



This is a digital copy of a book that was preserved for generations on library shelves before it was carefully scanned by Google as part of a project to make the world's books discoverable online.

It has survived long enough for the copyright to expire and the book to enter the public domain. A public domain book is one that was never subject to copyright or whose legal copyright term has expired. Whether a book is in the public domain may vary country to country. Public domain books are our gateways to the past, representing a wealth of history, culture and knowledge that's often difficult to discover.

Marks, notations and other marginalia present in the original volume will appear in this file - a reminder of this book's long journey from the publisher to a library and finally to you.

### Usage guidelines

Google is proud to partner with libraries to digitize public domain materials and make them widely accessible. Public domain books belong to the public and we are merely their custodians. Nevertheless, this work is expensive, so in order to keep providing this resource, we have taken steps to prevent abuse by commercial parties, including placing technical restrictions on automated querying.

We also ask that you:

- + *Make non-commercial use of the files* We designed Google Book Search for use by individuals, and we request that you use these files for personal, non-commercial purposes.
- + *Refrain from automated querying* Do not send automated queries of any sort to Google's system: If you are conducting research on machine translation, optical character recognition or other areas where access to a large amount of text is helpful, please contact us. We encourage the use of public domain materials for these purposes and may be able to help.
- + *Maintain attribution* The Google "watermark" you see on each file is essential for informing people about this project and helping them find additional materials through Google Book Search. Please do not remove it.
- + *Keep it legal* Whatever your use, remember that you are responsible for ensuring that what you are doing is legal. Do not assume that just because we believe a book is in the public domain for users in the United States, that the work is also in the public domain for users in other countries. Whether a book is still in copyright varies from country to country, and we can't offer guidance on whether any specific use of any specific book is allowed. Please do not assume that a book's appearance in Google Book Search means it can be used in any manner anywhere in the world. Copyright infringement liability can be quite severe.

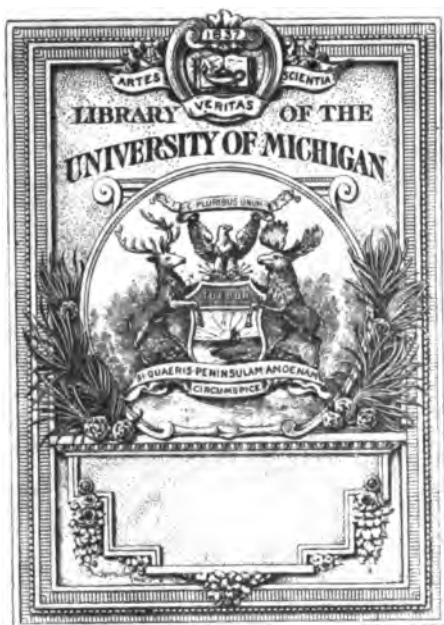
### About Google Book Search

Google's mission is to organize the world's information and to make it universally accessible and useful. Google Book Search helps readers discover the world's books while helping authors and publishers reach new audiences. You can search through the full text of this book on the web at <http://books.google.com/>

BUHR B



a39015 00015541 9b



SCIENCE LIBRARY

QD

905

.B37

200.



# **ELEMENTARY CRYSTALLOGRAPHY**

Published by the  
**McGraw-Hill Book Company**  
New York

Successors to the Book Departments of the  
McGraw Publishing Company      Hill Publishing Company

Publishers of Books for  
Electrical World      The Engineering and Mining Journal  
The Engineering Record      Power and The Engineer  
Electric Railway Journal      American Machinist

# ELEMENTARY CRYSTALLOGRAPHY

BEING PART ONE OF

GENERAL MINERALOGY

BY

W. S. BAYLEY, Ph. D.

ASSOCIATE PROFESSOR OF MINERALOGY AND ECONOMIC GEOLOGY  
UNIVERSITY OF ILLINOIS.

McGRAW-HILL BOOK COMPANY

239 WEST 39TH STREET, NEW YORK

6 BOUVERIE STREET, LONDON, E. C.

1910



COPYRIGHT, 1910  
BY THE  
MCGRAW-HILL BOOK COMPANY

*Printed and Electrotyped by  
The Maple Press  
York, Pa.*

0751055

## PREFACE.

---

The material in the following pages was originally intended to be the introductory portion of a text-book on Mineralogy. Because of the increasing interest in Crystallography on the part of chemists and others, however, it has been finally decided to offer it as an independent volume, in the hope that it might find a wider use in this form than if it were merely a portion of a larger volume which would appeal mainly to mineralogists. It is intended that it shall be followed in the near future by a second volume which will deal with minerals—the two taken together being expected to constitute an elementary text-book on Mineralogy.

The present volume does not pretend to be a treatise on Crystallography. It is merely a guide for those attempting to gain some insight into the fundamental principles underlying the science. It is the direct result of the need felt by the author in his own classes for a simple discussion of crystals and a series of simple statements of crystallographic conceptions. It is not intended to be a description of crystal forms, nor an illustrated list of crystallographic symbols. It is hoped that it may serve as an aid in teaching, but is not expected to serve as a reference book. Consequently some of the facts usually contained in text-books on Crystallography will not be found in its pages. Whether the material retained has always been chosen with wisdom must be left to the judgment of its readers. It was written for students of Mineralogy, Chemistry, and Physics. The mineralogist has always used crystallography as an efficient aid in his study of minerals. The chemist is beginning to appreciate its value in his work. The physicist will before long discover that it is absolutely necessary to employ its methods

in studying the physical properties of many of the substances with which he has to deal before he may hope to reach comparable results.

Because of the use to which the book is to be put, the author has assumed but little preliminary knowledge of mathematics, physics, and chemistry on the part of its users. Technical terms have been employed as rarely as possible, consistent with clearness, and only those topics are discussed which are necessary to the understanding of the nature of crystals. The discussion of optical phenomena has been reduced to a minimum, because of their complex nature, and the difficulty of observing them without a supply of expensive apparatus.

The choice of the Naumann System of parameter symbols in preference to the more elegant Miller System of indices has been made with deliberateness, solely because of the greater ease with which the Naumann symbols are comprehended by students who approach the study of crystals for the first time. The indices, however, are indicated for all the forms described or pictured.

There is no claim that any portion of the volume is original, except perhaps its method of developing the discussion. The illustrations have been collected from various sources. A few are original. Many of the figures illustrating symmetry were copied from Kraus's *Essentials of Crystallography*. Other figures have been taken from the text-books of Williams, Groth and Linck. When possible to do so, their sources have been indicated.

The writer is under obligations to Dr. R. M. Bagg, Jr., for photographs of some of the minerals illustrated in the text and to the publishers for their painstaking efforts to give the volume an attractive dress.

# CONTENTS.

---

Introduction.	Mineralogy: Its Object, History, and Division into Branches . . . . .	ix
---------------	--	----

## PART I.

### GEOMETRICAL CRYSTALLOGRAPHY.

CHAPTER	I.	General Facts of Crystallography . . . . .	3
CHAPTER	II.	The Law of the Constancy of Interfacial Angles . . . . .	14
CHAPTER	III.	The Law of Simple Mathematical Ratios . . . . .	17
CHAPTER	IV.	Symmetry . . . . .	25
CHAPTER	V.	The Isometric, or Regular, System—Holo- hedral Division . . . . .	31
CHAPTER	VI.	Partial Forms—Hemihedrism and Tetartohe- drism of the Isometric System . . . . .	41
CHAPTER	VII.	The Hexagonal System . . . . .	54
		Holohedral Division . . . . .	60
		Hemihedral Division . . . . .	69
		Tetartohedral Division . . . . .	82
CHAPTER	VIII.	The Tetragonal System . . . . .	87
		Holohedral Division . . . . .	87
		Hemihedral Division . . . . .	95
CHAPTER	IX.	The Orthorhombic System . . . . .	102
		Holohedral Division . . . . .	105
		Hemihedral Division . . . . .	112
CHAPTER	X.	The Monoclinic System . . . . .	115
		Holohedral Division . . . . .	117
		Hemihedral Division . . . . .	121
CHAPTER	XI.	The Triclinic System . . . . .	125
CHAPTER	XII.	Crystal Imperfections . . . . .	132
CHAPTER	XIII.	Crystal Aggregates . . . . .	143
CHAPTER	XIV.	Amorphous Substances and Pseudomorphs . . . . .	160
CHAPTER	XV.	Crystal Projection . . . . .	164

## PART II.

## PHYSICAL CRYSTALLOGRAPHY.

CHAPTER XVI.	Introduction: Physical Agencies and Physical Symmetry . . . . .	177
CHAPTER XVII.	Mechanical Properties of Crystals . . . . .	180
CHAPTER XVIII.	Optical Properties of Crystals . . . . .	196
CHAPTER XIX.	Thermal Properties of Crystals . . . . .	209
CHAPTER XX.	Electrical and Magnetic Properties of Crystals. . . . .	214

## PART III.

## CHEMICAL CRYSTALLOGRAPHY.

CHAPTER XXI.	Solution and Etched Figures. . . . .	219
CHAPTER XXII.	Isomorphism and Polymorphism . . . . .	227

## INTRODUCTION.

---

### MINERALOGY: ITS OBJECT, HISTORY, AND DIVISIONS INTO BRANCHES.

**The Purpose of Mineralogy** is the study of the solid, homogeneous, inorganic compounds occurring in nature, either as the constituents of rock masses or as the components of veins, which are the fillings of chinks or fissures in the earth's crust.

The science is closely related to Chemistry, since the objects of its study are definite chemical compounds, all of whose properties are believed to be determined by their composition, and to Geology, because their aggregations make up by far the greater portion of the earth's crust—the rocks.

The principles made use of in the study of minerals are not peculiar to the science of Mineralogy. Since minerals are definite chemical compounds they must be studied in part by chemical methods. The properties by which they are known are partly geometrical and partly physical, and these must be investigated by mathematical and physical processes. Mineralogy is thus not a pure science with methods of study peculiar to itself. It is a mixed science. Its problems are attacked by methods that are borrowed from other sciences and applied to minerals for the purpose of discovering their nature.

**Distinction between Minerals and Rocks.**—A *mineral* is any definite inorganic compound, either solid or liquid, occurring in nature. It is not the direct result of life processes nor the product of man's experiments.

A *rock* is a definite portion of the earth's crust irrespective of its composition. It is usually a mass of sufficient size to be regarded as an architectural unit in the structure known as the earth's crust. It does not necessarily possess a definite chemical composition, nor is it usually homogeneous. It may be an aggre-

gate of minerals of the same kind, as limestone, which is composed of grains of the mineral calcite ( $\text{CaCO}_3$ ), or it may be a mixture of minerals of different kinds, as granite, which consists essentially of feldspar, (a potassium-aluminium silicate), and quartz ( $\text{SiO}_2$ ), or it may be an accumulation of organic substances, like coal, or a mass of undifferentiated glass.

Different portions of the same rock when analyzed will usually give different results, and in only the few rare instances where a single mineral may constitute its entire mass can its composition be represented by a simple chemical formula.

According to the definitions given, water ( $\text{H}_2\text{O}$ ) is a mineral, as are also quartz ( $\text{SiO}_2$ ) magnetic iron ore ( $\text{Fe}_3\text{O}_4$ ), diamond (C), etc. Coal, amber, and wood are not minerals, because they are the products of organic processes. Granite and obsidian (volcanic glass) are not minerals because they do not possess definite chemical compositions. Tin is not a mineral because it is the product of man's work, while natural tin oxide, the ore from which the metal is obtained, is the mineral cassiterite or tin stone. The copper that occurs as native copper is a mineral, while the copper obtained by smelting the sulphide is an artificial product.

**History of Mineralogy.**—Although the study of the metals and of gems dates back certainly to the time of Aristotle (384 B. C.), the scientific study of the less valuable minerals began with Haüy (1743), a famous French teacher, who discovered that all minerals possess characteristic forms which may be deduced in some cases from their physical properties. In the last half of the eighteenth century and in the early years of the nineteenth century, Werner (1750) and Weiss (1808), in Germany, described in great detail a large number of minerals, and the latter author developed a method of classifying and representing their forms. From this time until the present day Germans have been the most earnest students of Mineralogy in all its branches. France, Great Britain, and Italy have many devoted mineralogists, while North America has produced quite a number who have acquired world-wide reputations.

The science has recently become so broad that no one pretends

to be versed in all its branches. There are descriptive mineralogists, physical mineralogists, optical mineralogists, mathematical mineralogists, and chemical mineralogists, the members of each class devoting themselves to special phases of the science, but all being more or less thoroughly acquainted with the results of the work in all its other phases.

**Divisions of the Subject.**—Since Mineralogy is such a broad science, dealing as it does with the geometrical, physical, and chemical properties of minerals, it has been found convenient to divide it into branches, each one of which is devoted to the study of some special class of properties.

The fundamental basis of mineralogy is chemistry. All minerals are definite chemical compounds, the properties of each one of which are directly dependent upon their chemical composition and structure. Thus Chemical Mineralogy is one branch of the subject, and upon it all the other branches are founded.

When different chemical compounds are studied it is soon discovered that they vary in color, hardness, ease of fusibility, etc., i.e., in their physical properties. We therefore recognize a branch of mineralogy that is devoted to the investigation of the physical properties of minerals—Physical Mineralogy.

Again, careful scrutiny of these same compounds soon reveals the fact that individual minerals are characterized by definite and distinctive forms by which they may be recognized. Thus a piece of *quartz* ( $\text{SiO}_2$ ) may often be distinguished from a piece of *calcite* ( $\text{CaCO}_3$ ) simply by its external form. Morphological Mineralogy deals with the forms of minerals. But distinctive forms are not confined to minerals. They are possessed likewise by other substances which have been produced under conditions analogous to those under which minerals are formed. They are characteristic of nearly all solid substances that have been sublimed from gases or vapors or have been precipitated from solutions. The study of such forms from the general point of view is known as *Crystallography*. Further, bodies exhibiting these definite geometrical forms are also characterized by certain well-defined physical properties which



are closely related to the forms exhibited. Consequently, Crystallography is naturally divisible into two branches, viz., Geometrical Crystallography and Physical Crystallography.

When minerals are studied with respect to their individual properties, we have Descriptive Mineralogy.

After becoming acquainted with the characteristics of different minerals it often becomes necessary to make use of these characteristics in determining the nature of an unknown substance. This application of our knowledge concerning the properties of known minerals to the discovery of the nature of an unknown mineral is called Determinative Mineralogy. In Chemistry it corresponds to Analytical Chemistry.

The present volume deals only with the elements of Geometrical and Physical Crystallography. A later volume will be devoted to a discussion of the general characteristics of minerals and descriptions of the features of the most common kinds.

PART I.  
GEOMETRICAL CRYSTALLOGRAPHY.



## CHAPTER I.

### GENERAL FACTS OF CRYSTALLOGRAPHY.

**Forms Assumed by Substances.**—The forms assumed by substances through the agency of natural forces are of two classes: (1) those produced by the action of internal molecular forces and (2) those produced by external agencies. The first depend upon the nature of the material composing the substance, and are known as *Idiomorphic* forms; the second are to a large extent independent of the nature of the substance, but are determined by the nature of the mechanical agents acting upon it and the directions along which they act, or by the methods by which it was produced. They are accidental forms. Since the forms



FIG. 1.—Crystals of calcite attached at one end.



FIG. 2.—Stalactite of calcite.

of this class are not directly dependent upon the nature of the bodies exhibiting them, they cannot be classified and studied systematically.

Figure 1 illustrates an idiomorphic form (a group of crystals of calcite ( $\text{CaCO}_3$ )), and figure 2, an accidental form (a stalactite of the same substance). The shape of the crystals (Fig. 1) is due to the fact that they are *crystals of calcite*. Crystals of other substances have not the same shape as this one. The shape of the stalactite (Fig. 2) is due to the fact that the calcite of which it is

composed was deposited from dripping water and not to the fact that its substance is calcite. There are stalactites of many substances not at all similar to calcite. Among other forms of the second class may be mentioned those of mineral pebbles, fragments, cut stone, like the sets in rings, etc., all of which were produced by forces which had their origin outside the substance.

**Idiomorphic Forms.**—On the contrary, the idiomorphic forms are directly dependent upon the nature of the substance of which they are characteristic. Calcite ( $\text{CaCO}_3$ ) always possesses definite forms that are distinctive for this mineral; that is, exactly the same form is never seen in other minerals. Quartz ( $\text{SiO}_2$ ) also has its distinctive forms, which are peculiar to itself. So with many other substances.

Occasionally several substances which are apparently identical in chemical composition may have different idiomorphic forms, as is the case with calcium carbonate ( $\text{CaCO}_3$ ). The apparent identity of such substances is, however, believed not to be a real identity. There are two calcium carbonates which differ from one another in the structure of their chemical molecule or in the way in which their molecules are united. They are, therefore, different chemical compounds. These calcium carbonates possess different idiomorphic forms, as we should expect of them, and, therefore, they are given different names. One of them is the mineral *calcite* and the other the mineral *aragonite*.

*Idiomorphic forms* may be defined as *those which are original to the body on which they occur, and which are essential to its substance.*

**Molecules and Crystal Particles.**—All bodies are believed to be made up of tiny particles called molecules, which vary in size and weight for different substances. They are so very small that the most powerful microscope will not detect them. In a single cubic inch of a cold gas like nitrogen they are present to the number of about 110 billion billions. These molecules are so small that it would take about 55,000,000 to make a row one inch long. Nevertheless, if all that are present in a cubic inch of gas were placed in a row, touching one another, they would form a line 32,000,000 miles long. The molecules are always in motion,

but they are supposed not to touch each other even in the hardest and densest bodies. They are separated from one another by tiny spaces.

When a body passes from the liquid or gaseous state to the solid form, the little molecules tend to arrange themselves into definite groupings, which are the crystal particles. These are built up by the addition of other particles until they become large enough to be seen first by the microscope, then by the naked eye, until finally they reach a size that may be measured by inches or even feet. All the little particles of a body built up in this way are constructed on the same plan, so that every portion of the body has the same structure. If we should take bits from its different parts and could magnify them sufficiently to render their component molecules visible, we would find these little bodies all arranged in a definite manner, which would be the same for every fragment. If we could examine other bodies in the same way, we should find that they, too, are composed of molecules arranged in certain definite groupings; but, in many cases, we should discover these groupings to be different from those of the first body studied.

While we cannot see the method of grouping of the particles, we can infer something of its character from the phenomena presented by different kinds of bodies. From the effects produced we can study the groupings themselves, and thus can gain an insight into the internal structure of the bodies which they compose.

From a careful consideration of the subject from all points of view it is concluded that the arrangement of the molecules in any solidifying body is dependent almost exclusively upon the nature of the little molecules themselves. It is believed that the power to build up well-defined and characteristic groupings is a property of the molecules, just as the power of chemical combination between the atoms is an inherent property of the atoms. But the chemical composition of every homogeneous body is the same as the chemical composition of its component molecules, hence we may infer that the form assumed by a body is dependent upon its composition.

In order to explain the peculiarities of a body in its different states it seems necessary to assume that *every molecule possesses attractive powers* by which it tends to pull toward itself other molecules of the same kind, and that *at the same time it is endowed with the impulse to move in straight lines*. These two properties which are supposed to be inherent in every molecule tend to counteract one another in part. When the tendency to move away from its neighbors (the repulsive tendency) predominates in the molecules composing a body, that body is said to be a *gas*. When the attractive tendency overcomes the repulsive one, the body is a *liquid* or a *solid*. The distinction between the solid and the liquid states depends upon other considerations—such, perhaps, as the friction between their constituent molecules. The two pass gradually into one another so that it is often difficult to discriminate between them. A *body* is usually said to be a *liquid* if it assumes a *spherical form when floating in a medium with which it does not mix*.

**Crystalline Bodies and Crystals.**—Not all solid substances are constructed of regularly arranged molecules. In some the molecules possess no regularity of arrangement, so far as we can learn. This class includes such bodies as glasses, jellies, etc., which are spoken of as *amorphous* or *colloidal*. Other substances, like the tissues of plants and animals, are constructed of cells regularly built up, but this regularity is not the result of the piling up of the molecules in a definite manner in consequence of any property inherent in them, but is the result of the life processes of the animals or plants that make the cells. The molecules in the cells are not regularly and definitely arranged, although the shapes of the little cells themselves may all be alike. In other words, while the tissue is composed of regularly arranged cells, the materials composing the cells may be amorphous.

*When a solid or fluid body is composed throughout of molecules arranged according to some definite plan, it is said to be crystalline; when composed of molecules arranged helter-skelter, i.e., without any definite plan, it is said to be amorphous or colloidal.*

The distinction between an amorphous body of definite form and a crystalline body may be easily shown with the aid of a few

buckshot. If a layer of these be built into a triangle, as illustrated in figure 3 *a*, and on this layer a second layer be placed in the manner indicated, and upon this a third layer, and so on, we finally obtain a pile of shot constructed of regularly arranged components (Fig. 3 *b*). The shape of the pile depends upon the plan of the foundation triangle and upon the manner in which the successive layers are piled upon the first one. If we let the shot represent molecules, the pile represents a crystalline body with a regular internal structure. On the other hand, we may tumble the shot indiscriminately into several little boxes all made exactly alike. The masses of shot within the boxes would all

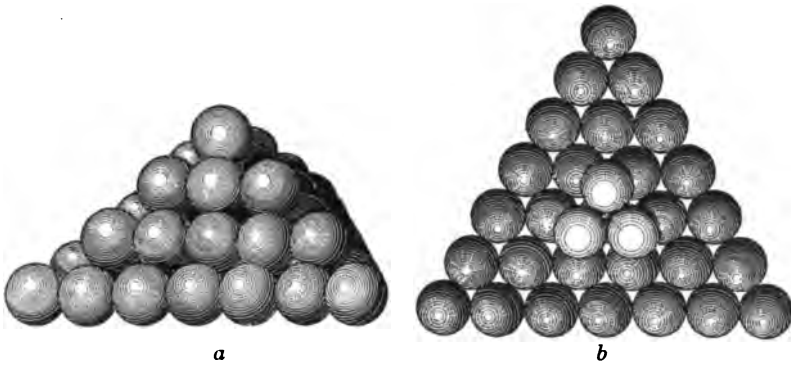


FIG. 3.

have the same shapes—the shape of the boxes—but there would be no regularity in the arrangement of the shot. The contents of the boxes illustrate the structure of an amorphous body.

Since crystalline bodies exhibit their physical properties in different degrees along different directions, a *second assumption with regard to molecules* is necessary, viz., *that their activities are oriented*. In other words, the lines along which their activities are exerted are definitely arranged with respect to one another. When we are dealing with an amorphous body in which there is no regularity in the arrangement of its molecules, there is, as a consequence, no regularity in the arrangement of the directions along which their powers act; consequently such bodies possess similar properties in parallel directions. Thus a sphere of glass, which



is amorphous, remains a sphere under all conditions of temperature, i.e., it expands equally in all directions. On the other hand, a sphere of quartz, which is crystalline, loses its spherical form when the temperature rises or falls and becomes ellipsoidal, i.e., it expands and contracts to a greater degree in one direction than in others. Its physical properties exhibit a definite arrangement.

**Crystals.**—When the little molecules of crystalline bodies are given the opportunity and the time to arrange themselves in accordance with the plan that suits them best, they build up structures usually bounded by planes, which vary in number and in inclination to one another according to certain definite laws. *A homogeneous crystalline body thus bounded by planes is called a crystal, or a crystal individual. Or, since the bounding planes are the result of the internal structure, a crystal may be defined as a homogeneous body bounded by a polyhedron that is idiomorphic.*

The term homogeneous is introduced into the definition because it sometimes happens that a single body bounded by plane faces consists of several parts united in such a way that, while the composition and arrangement of the molecules in all portions of each part is the same, the composition and arrangement in the different parts is different. The term homogeneous signifies that all parts of the individual are like all other parts.

Although the great majority of crystalline bodies are solid, there are some liquids which under proper conditions exhibit a definite internal structure which is recognizable in polarized light (see page 196) and which, when in the form of drops, show a tendency to bound themselves by plane surfaces. These are known as *liquid crystals*. They do not possess the sharply defined forms of solid crystals, but are distinctly bounded by surfaces that are nearly plane and that intersect in edges (see Fig. 4).

**Crystallization.**—The process by which crystals and crystalline bodies are formed is known as *crystallization*. No satisfactory explanation of the force of crystallization has yet been proposed. It is known that some substances possess this power to a much greater degree than others, and in some (the colloids) it appears to be entirely lacking. In the latter cases, however,

the power may still exist, but if so it is so weak that it can express itself only under the most favorable conditions.

**Crystallography.**—The external forms of crystallized bodies are their most striking peculiarities, consequently they have been studied longer than their other features. They were at first supposed to be their most essential characteristics, and hence their study was early named *crystallography*. It is, however, now well recognized that the forms are only one of the methods through which the molecular structure of a crystallized body expresses itself. The term as now used includes not only the



FIG. 4.—Photograph of liquid crystals with rounded edges.  
(After O. Lehmann.)

study of the forms of crystals, but also the study of all their other characteristic properties. Sometimes the former is distinguished as *geometrical crystallography* and the latter as *physical* and *chemical crystallography*. Crystallography includes the study of the forms assumed by all crystallized bodies, whether these bodies occur as minerals or whether they exist only as the products of the laboratory. When applied to mineral substances it is often called "*Morphological Mineralogy*."

**Laws of Crystallography.**—Almost as soon as the forms of crystals began to be studied carefully it was discovered that the planes which comprise them are fixed in position and that their relations to one another are so well-defined that they may be expressed mathematically. The crystallographic laws thus discovered are very simple and are the foundation of all modern crystallography. They are three in number, as follows:

1. The law of the constancy of the interfacial angles on all crystals of the same substance.



marked in a third. The zonal axis of the third zone is the line AB, that of the first zone is CD, and that of the second zone EF, provided the lines are regarded as passing through the center of the crystal.\*

**Measurement of Interfacial Angles.**—In order to determine the elements of crystals it is necessary that the values of their interfacial angles should be known, as the inclination of the planes to one another varies with the values of these angles, or, to put

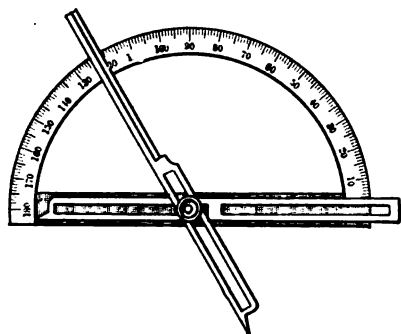


FIG. 7.—Contact goniometer.

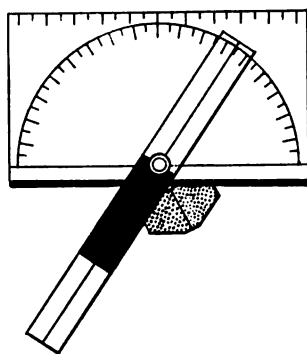


FIG. 8.—Simple form of contact goniometer.

the case more logically, the values of the interfacial angles vary with the inclination of the planes including them.

The instruments with which the interfacial angles are measured are known as *goniometers*. The simplest form is the *contact goniometer* (Fig. 7). This form consists of a graduated arc and two detachable arms, one of which revolves about a pivot common to both. These arms are removed from the arc and applied to the two faces of the crystal whose angles are to be measured, care being taken to hold them perpendicular to the interfacial edge, and at the same time to press them firmly

\* Before entering further upon the study of crystals the student should familiarize himself with the terms defined, by applying them to the proper parts of a few simple crystals or crystal models. First the edges, the interfacial and the crystal angles should be pointed out, and then all the zones occurring on a few more complicated models should be determined. A very useful and cheap set of 60 crystal models in wood may be obtained from dealers in minerals. Other and more comprehensive sets may be obtained from Dr. F. Krantz, Bonn on Rhine, Germany, at an average cost of 45 cents for each model.

against the faces. The screw holding the arms is then set, and the arms themselves are replaced on the arc, when the value of the angle desired is indicated by the graduation. Measurements made with the contact goniometer are reliable to about half a degree.

Other forms of the contact goniometer are even simpler than the one described. Their construction, however, is so similar to that of the one illustrated that they demand no description (see Fig. 8).

A more accurate but much more complicated instrument is the *reflection goniometer* (Fig. 9). Its use depends upon the prin-

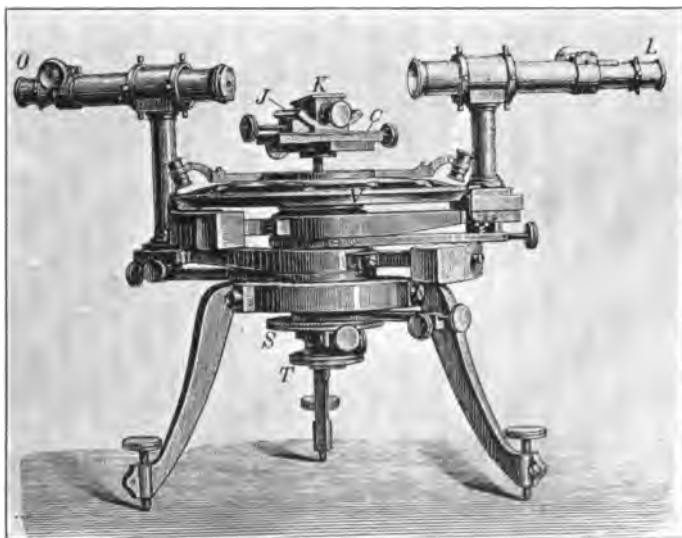


FIG. 9.—Reflection goniometer.

ciple that a ray of light falling upon two planes in parallel position is reflected from them in parallel directions. The mechanism of the instrument is such as to give a fixed ray of light (through L in the figure), a means of detecting its reflection (O), and an arrangement for so placing a crystal that its contiguous planes may be brought successively into a given position (K). By the use of a well-made instrument the angle through which the crystal must be turned in order that its faces shall give

parallel reflections can be read off to within a fraction of a minute of arc on the graduated circle V. This angle is the supplement of the interfacial angle.

Let AOC (in Fig. 10) be the interfacial angle to be measured and EP a ray of light falling upon the face AO. It will be reflected along the line PG. In order that the same ray shall be reflected from the face CO in the same direction this face must be brought into the position OC', when OA will have the position OA'. But in order to bring OC into the position OC' the crystal must be turned through the angle COC', which is the supplement of the interfacial angle COA.

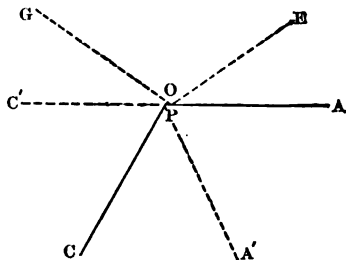


FIG. 10.

#### **Importance of Accurate Measurements of Interfacial Angles.**

—The importance of determining accurately the values of interfacial angles is realized when we remember that only

those planes that are parallel to each other can be equally inclined to a given plane, hence planes that make different angles with a given plane cannot be parallel to one another. Moreover, since in crystallography planes are distinguished from each other by their inclinations to standard planes, it follows that planes that make different angles with a given plane are different. It further follows that when two planes on one crystal have a different interfacial angle than that possessed by two planes on a second crystal, the planes on the two crystals cannot be the same.

The values of interfacial angles may thus serve to distinguish between substances that are otherwise apparently identical. For instance, certain light brown crystals of calcite ( $\text{CaCO}_3$ ) bear a strong resemblance to certain crystals of light brown siderite ( $\text{FeCO}_3$ ). If the crystals are six-sided (rhombohedrons) they contain corresponding interfacial angles, which on the calcite have a value of  $106^\circ 15'$ , and on the siderite  $107^\circ$ . By careful measurement this difference may be recognized and the two minerals may be distinguished.

## CHAPTER II.

### THE LAW OF THE CONSTANCY OF INTERFACIAL ANGLES.

**Statement of the Law.**—The observation and comparison of crystals have shown that *however much the crystals of a given substance may vary in size and shape, their corresponding interfacial angles are the same in value, provided they be measured under the same conditions.*

**Significance of the Law.**—This law expresses the fact that it is not the general shape, or *habit*, of a crystal that is its most important external feature, but the inclination of the planes by which it is bounded. Two crystals may look very much alike, i.e., they may be similar in habit, but if the inclinations of their

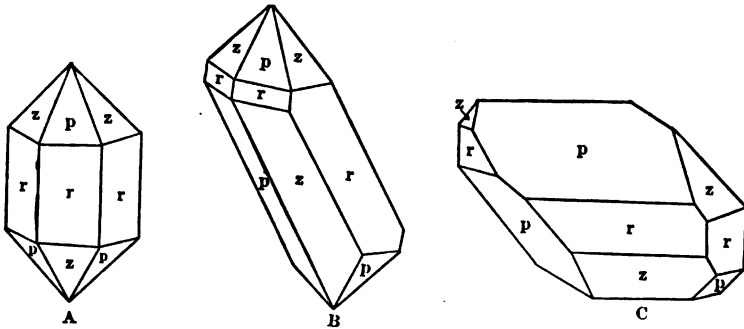


FIG. 11.—Three crystals bounded by the same planes, but with different habits.

corresponding faces are different the crystals themselves are made up of different crystallographic elements. On the other hand, if two crystals that are apparently entirely unlike each other are bounded by planes with their corresponding interfacial angles equal in value, the corresponding crystal faces are regarded as the same faces and the crystals are crystallographically identical.

The crystals A, B, and C represented in the subjoined figure (Fig. 11) are very different in habit, and yet they are bounded

by the same planes, since the interfacial angles on A are identical in value with the corresponding angles on B and C (between planes marked with the same letters).

The condition imposed by the law, viz., that the crystals shall "be measured under the same conditions," is necessitated by the fact that crystals deport themselves toward physical agencies in a different manner from non-crystallized substances. Under the influence of changes in temperature, for instance, they may expand or contract differently in different directions, and the inclinations of their faces will vary accordingly. Even the same crystal will yield different values for the same interfacial angle when this is measured at different temperatures, hence in comparing the angles on different crystals it is necessary to know at what temperature they were measured in order to decide whether or not the faces including the angles are the same.

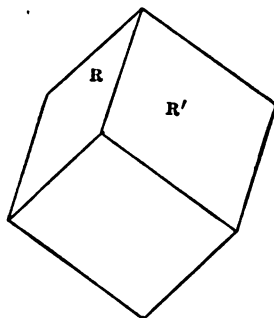


FIG. 12.

The expression of the law further indicates that it applies only to crystals of the same substance. From this we may rightly infer that crystals of different substances possess different interfacial angles even though bounded by similar crystallographic planes. As a fact, except among the bodies crystallizing in what is known as the regular or isometric system, and with the exception of a very few angles on other crystallized bodies, the crystals of different substances possess different interfacial angles, which, however, are so distinctive for each substance that they may often serve as a means for its identification. The values of the interfacial angles are therefore characteristic for different substances. They must consequently vary according as the chemical composition of the substance varies. When the direction and amount of the variation caused in the value of any angle by the admixture of a certain chemical element into the molecule is known the quantity of this element introduced may frequently be determined by the difference in the value of the



angle observed and that of the corresponding angle in the pure substance (see pages 231-232).

In the mineral calcite ( $\text{CaCO}_3$ ) the angle between the faces R and R' is  $74^\circ 55'$  (Fig. 12), while in magnesite ( $\text{MgCO}_3$ ), which crystallizes like calcite, it is  $72^\circ 31'$ . In the mineral dolomite ( $\text{MgCaCO}_3$ ), which crystallizes in the same shape, the corresponding angle is  $73^\circ 45'$ . This is between that of calcite and that of magnesite.

**Morphotropism.**—The influence which the introduction of an element or a group of elements into a compound, or their abstraction from a compound, exerts upon the values of the interfacial angles is known as *morphotropism*. The action itself is known as morphotropic action. The morphotropic action of Mg in calcite, for instance, is to decrease the value of the angle  $R \wedge R'$ .

**Corollary to the Law.**—Since the interfacial angles are the important elements in defining the nature of crystals, and not the sizes or shapes of the faces or their distances from the centers of the crystals, and since the only planes that can make equal interfacial angles with a given plane are those that are parallel to each other, it follows that parallel faces in crystallography may be regarded as the same face, situated at different distances from the center of the crystal.

Thus a small cube and a large cube are crystallographically identical forms, which differ from each other only in the distances of their bounding planes from the centers of the respective forms.

## CHAPTER III.

### THE LAW OF SIMPLE MATHEMATICAL RATIOS.

#### **The Designation of the Positions of Planes in Space.—**

The position of a plane in space may be defined by referring it to a system of three lines intersecting at a common point, just as the position of a line on a plane may be defined by referring it to two other intersecting lines.

Planes on crystals are referred to a system of imaginary lines, known as *axes*, which are supposed to intersect at the center of the crystal. The positions and inclinations of the planes are

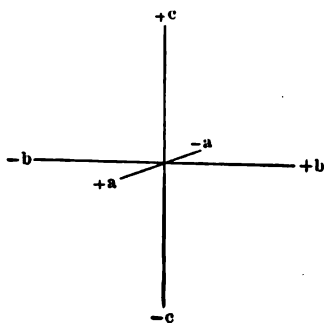


FIG. 13.—System of axes.

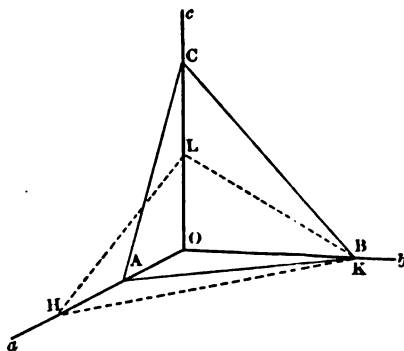


FIG. 14.

defined by expressing the relations between the distances at which they cut the axes, measured from the point of their common intersection.

In figure 13 is illustrated a scheme of axes to which the planes on certain crystals may be referred. In order to locate the planes accurately each axis is designated by a letter, and the two ends of each axis are distinguished by different signs. The positions of planes are thus easily indicated by stating the relative distances at which they cut the three axes. The plane ABC (Fig. 14) may be defined as a plane cutting the *a* axis at the distance OA, the *b*

axis at OB, and the  $c$  axis at OC. The plane HKL may likewise be defined as a plane cutting  $a$  at OH,  $b$  at OK, and  $c$  at OL.

**Parameters.**—In all measurements of distance a standard length is assumed, in terms of which the result of the measurement is expressed. In ordinary practice the standards made use of are the inch, foot, yard, mile, or metre, kilometre, etc. In crystallography the standard lengths are those distances at which a selected plane, called the groundplane or *groundform*, cuts, or intercepts, the axes. These distances are taken as units, and the distances at which other planes on the same crystal intercept the corresponding axes are expressed in terms of these units. The resulting expressions are known as the *parameters of the plane*. If ABC in figure 14 is taken as a groundplane, then its intercepts OA, OB, and OC on the three axes are the unit lengths on these axes. The plane HKL intercepts the axes at OH, OK, and OL. If  $OH = 2\ OA$ ;  $OK = OB$ , and  $OL = \frac{1}{2}\ OC$ , then the position of HKL may be expressed by stating that its intercept on  $a$  is twice the unit length on  $a$ , on  $b$  the intercept is the unit on  $b$ , and on  $c$  the intercept is  $\frac{1}{2}$  the unit on  $c$ , or, briefly, the symbol representing the plane is  $2a : b : \frac{1}{2}c$ , in which  $a$ ,  $b$ , and  $c$  represent the unit lengths on the three axes. The values 2, 1, and  $\frac{1}{2}$  are the parameters of the plane on the axes  $a$ ,  $b$ ,  $c$ , respectively.

**General Symbol for a Plane.**—The most general symbol for any plane is  $ma : nb : pc$ , in which  $m$ ,  $n$ , and  $p$  represent definite values other than one. A plane with this symbol cuts the three axes at different distances, and no one of the distances is the unity for that axis. But parallel planes in crystallography are regarded as the same plane (see page 16), hence one may imagine them moved parallel to themselves without affecting their relations to other planes. The plane  $ma$ ,  $nb$ ,  $pc$ , may be shifted in position until it cuts  $b$  at unity. It will then cut  $a$  at  $\frac{m}{n}a$ , and  $c$  at  $\frac{p}{n}c$ , provided its new position is parallel to its original one, and the symbol of the plane will become  $m'a : b : p'c$ . The parameters are thus ratios between the intercepts of a plane on the axes and the unity distances on the same axes, when the plane is moved parallel to itself until it intercepts one of the axes at a unity (or

standard) distance. By this method corresponding planes of similar crystals are represented by the same symbol whether the crystal be small or large. *The parameters are ratios and not absolute distances.*

One of the theorems of solid geometry states that if two parallel planes are cut by any third plane, the intersections are parallel.

Let ABC (Fig. 15) be a plane cutting the axes  $a$ ,  $b$ , and  $c$  at A, B, and C, respectively, and let  $OA=m$ ,  $OB=n$ , and  $OC=p$ . Let A', B', C' be the position of this plane when it is moved parallel to itself until it cuts  $b$  at unity (B').

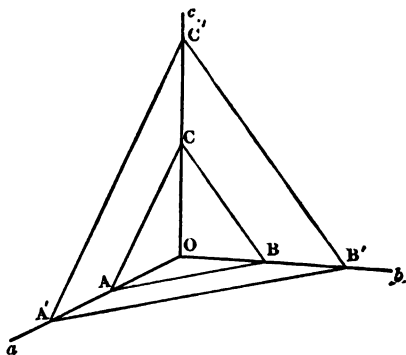


FIG. 15.

Pass a plane through the axes  $OC'$  ( $c$ ) and  $OB'$  ( $b$ ), then will the intersections of this plane with

ABC and A'B'C', or the lines BC and B'C', be parallel. The

triangles OBC and OB'C' are therefore similar, and  $OB : OB' :: OC : OC' = n : 1 :: p : OC'$ , and  $OC'$  (the intercept of A'B'C'

on  $c$ ) =  $\frac{p}{n}$ . In a similar way it can be proven that the new inter-

cept on  $a = \frac{m}{n}$ .

The values  $m$ ,  $n$ , and  $p$  of the first symbol are not the same as  $m'$ ,  $1$ , and  $p'$  of the second symbol, but the ratios between  $m$ ,  $n$ , and  $p$  are the same as those between  $m'$ ,  $1$ , and  $p'$ . This being the case, it is convenient to express the symbol of a plane in the form of a ratio, for then it becomes a general symbol for any plane parallel to a given plane. Thus  $m'a : b : p'c$  is the symbol for any plane cutting the three axes in the ratio  $m'$  times, once, and  $p'$  times the unity distances on  $a$ ,  $b$ , and  $c$ .

For instance, the symbol  $2a : b : 1/2c$  designates any plane that cuts the axes at distances that are in the ratio of  $2 : 1 : 1/2$  times the unities on the three axes. The symbols  $4a : 2b : c$  and  $a : 1/2b : 1/4c$  designate the same plane as does the symbol

$2a : b : 1/2c$  because the ratios between the parameters are the same in all cases. The plane  $4a : 2b : c$  may be regarded as being at twice the distance from the center of the crystal as is the plane  $2a : b : 1/2c$ , and the plane  $a : 1/2b : 1/4c$  at only half this distance; but all the planes make the same interfacial angles with the corresponding adjacent planes and hence are regarded as the same plane. If the three symbols express any difference in the planes it is simply with respect to their actual distance from the center of the crystals on which they are found; or, in other words, they express simply a difference in the size of the crystals. A cube is a cube, whether it measures  $1/10$  of an inch or 10 inches on an edge. Crystallography takes no account of the size of crystals; it deals only with the distribution and inclination of the planes that compose them.

**Crystallographic Notation.**—In discussing crystals and crystal planes it is necessary to make use of some method of representing them accurately and at the same time graphically. The principles upon which one system of notation is based are explained in the previous paragraph. This is known as the *system of parameters* or the *Weiss system*, after its inventor.

**The Weiss System of Notation.**—In this system the crystallographic axes are invariably written in the order  $a : b : c$ , and to these are prefixed the parameters, when these vary from unity. The symbol  $a : b : c$  without any parameters always refers to a plane of the groundform, that is, to a plane cutting the three axes at the distances assumed as the unities. The parallelism of a plane to a crystallographic axis is indicated by the sign of infinity,  $\infty$ , written in its proper place as a parameter. Thus  $\infty a : b : c$ , represents a plane parallel to the  $a$  axis and cutting  $b$  and  $c$  at unity.  $\infty a : \infty b : c$ , is a plane cutting  $c$  at unity, and at the same time parallel to  $a$  and  $b$ . Further, it is customary for simplicity's sake to make the parameter on either the  $a$  or the  $b$  axis unity, and to reduce the symbol accordingly. For instance, if the measurement of a crystal should yield  $42a : 6b : 12c$  as the symbol for a certain plane, this would be reduced to the form  $7a : b : 2c$ , which would become the symbol for the plane.

By reference to figure 13 (page 17) it will be noted that this

symbol stands for a plane in the upper right-hand division or octant of space included within the halves of the three axes. Corresponding planes in the other octants on the upper half of the crystal would be represented by  $-7a : b : 2c$ ;  $-7a : -b : 2c$ , and  $7a : -b : 2c$ . On the lower half of the crystal we should have the corresponding planes  $7a : b : -2c$ ;  $-7a : -b : -2c$ ;  $-7a : b : -2c$ , and  $7a : -b : -2c$ . Each symbol thus represents a different plane, the position of which on the crystal is indicated by the signs. Except with respect to the signs, all the symbols are alike. Hence, if the presence of one of the planes should necessitate the presence of all the others, then the signs could be omitted without affecting the significance of the symbol. It would then indicate not simply a single one of these corresponding planes, but any one of them, that is, it would become a general symbol for all. Upon further study it will be found that the presence of a given plane does necessitate the presence of one or more similar planes on the same crystal, and in this case the symbol of any one of the planes, written without signs, becomes the symbol for all of them.

**Crystal Forms.**—A crystal form is the sum of all the planes on a crystal that may be represented by symbols differing only in signs. The eight planes indicated in the last paragraph, and of which  $7a : b : 2c$  is one, constitute a form.

**Naumann's System of Notation.**—The Weiss system of notation has to do primarily with the *planes* comprising crystal forms. It is cumbersome and more or less indefinite, as a symbol may be not merely the symbol of a form, but the symbol of one of the faces of the form as well. Naumann has proposed a system of notation which is in principle the same as that of Weiss, but which has the advantage of representing forms while at the same time it is briefer than that of Weiss.

In this scheme the groundform is represented by the letters O or P, according as the unities on the three axes are equal or different in value, and the parameters of the planes are written before or after the symbol for the groundform, according as they refer to the  $c$  or to one of the other axes. One of the latter is

always made unity, and unity values are omitted from the symbol.

The symbol  $a : b : c$  becomes in the Naumann notation O or P,  $2a : b : 1/2c$  becomes  $1/2P_2$  or  $1/2O_2$ , and  $7a : b : 2c$  becomes  $2P_7$  or  $2O_7$ . The symbol  $a : 5b : 2c$  would become  $2P_5$  or  $2O_5$ ,  $a : b : 2c$  would be  $2P$  or  $2O$ ,  $a : 2b : c$  or  $2a : b : c$  would be  $P_2$  or  $O_2$ . The  $\infty P \infty$  in Naumann's notation is  $\infty a : b : \infty c$  or  $a : \infty b : \infty c$  of Weiss.  $\infty a : \infty b : c$  is represented by  $\infty P$ , for to make  $a$  or  $b$  unity one must divide by  $\infty$ , when we have  $\frac{\infty}{\infty}a : \frac{\infty}{\infty}b : \frac{1}{\infty}c$ , which is  $1a : 1b : \infty c$ .

**Dana's Notation.**—Prof. Dana has further simplified Naumann's system by substituting a hyphen for the initial O or P, and the letter i or I for the infinity sign  $\infty$ .  $1/2P_2$  of Naumann =  $1/2-2$  of Dana.  $2P_7 = 2-7$ ,  $\infty P_2 = i-2$ ,  $2P \infty = 2-i$ ,  $\infty P \infty = i-i$ . When the groundform exists alone it is represented by 1. Thus O or P of Naumann = 1 of Dana.

If only one parameter differs from unity it is written alone if it refers to  $c$ , as  $2P = 2$ ,  $P = 1$ ; but if it refers to  $a$  or  $b$ , it is preceded by 1, as  $P_2 = 1-2$ ,  $P \infty = 1-i$ .

**Statement of the Law of Simple Mathematical Ratios.**—The second law of crystallography may be expressed as follows: *Experience shows that only those planes occur on crystals whose parameters are either infinite or are small fractions or small even multiples of unity.* The ratio existing between them is simple.

Under certain circumstances of growth not yet clearly understood there occur on some crystals small planes with very large and sometimes irrational parameters. These are known as vicinal planes. They are probably produced by causes which are partly or wholly external in origin, and are, strictly speaking, not idiomorphic forms due to the crystallizing force inherent in the molecules of the crystallizing substance.

If this law is a correct statement of fact, the plane  $1.675 a : b : 3.272 c$  is crystallographically impossible. In the experience of crystallographers no plane with these parameters has been found, unless it may be in some of the vicinal forms.

**Practical Utility of the Law.**—The practical value of the

law is soon seen when we come to make use of it in determining the symbols of planes from the measurements of their interfacial angles. The calculation of the intercepts of a plane is apt to lead to results that vary slightly from simple fractions or small whole numbers, on account of the practical impossibility of determining accurately the true values of the interfacial angles because of the imperfections of our instruments and the irregularities in the surfaces of the crystal faces. Under these circumstances we confidently take the nearest simple fractions or the nearest whole numbers to represent the true intercepts of the plane under investigation. For example, if we should calculate the intercepts of a certain plane to be  $1.973 a : b : .508 c$ , we would not hesitate to write its symbol  $2a : b : 1/2 c$ .

**The Miller System of Notation.**—There is another system for designating crystal planes that is even more widely used than the system of Naumann or of Dana. Its advantage over these latter is due largely to the greater ease with which the symbols can be manipulated in mathematical discussions. Although very simple in principle, it is a little more difficult of comprehension than Naumann's system.

In the Miller system the symbol of a plane consists of its indices written in the order given above for the axes, viz.,  $a : b : c$ , and in the simplest form possible, without the use of fractions. *The indices are the reciprocals of the parameters.* The symbol  $3P_{3/2}$  of Naumann (when  $3/2$  refers to the  $a$  axis) represents a plane whose parameters for the different axes are  $3/2a : b : 3c$ . The reciprocals of these parameters are  $2/3 : 1 : 1/3$  taken in order. When cleared of fractions these become  $2 : 3 : 1$ , and the symbol of the plane, according to the Miller system, is  $2\ 3\ 1$ . The indices are always written in the same order and the symbol for the axes is omitted.

The transfer between the Naumann and the Miller symbols is easily effected if it is remembered that the latter are always written in the order  $a : b : c$  and that no fractions are employed. In the Naumann symbol, on the other hand, the parameter on  $c$  is written before the initial representing the groundform of the crystallized substance, except in the isometric system in which



there is no distinction made between the three axes. In transferring from the Naumann to the Miller system, then, it is only necessary to write the parameters in the  $a : b : c$  order, take their reciprocals and clear of fractions. To transfer in the opposite direction the reverse process is employed, the smaller of the parameters on  $a$  and  $b$  being made unity. When the parameters are obtained, only those not unity are represented, and that referring to the  $c$  axis is written in front of the initial that stands for the groundform.

Thus:

$$\begin{array}{ll}
 \frac{3}{2} P \tilde{3}^* = 3a : b : \frac{3}{2} c & \text{(parameters)} \\
 \frac{1}{3} \quad 1 \quad \frac{2}{3} & \text{(indices)} \\
 1 \quad 3 \quad 2 & \text{(Miller symbol)} \\
 3 P \tilde{3/2}^* = \frac{3}{2} a : b : 3c & \text{(parameters)} \\
 \frac{2}{3} \quad 1 \quad \frac{1}{3} & \text{(indices)} \\
 2 \quad 3 \quad 1 & \text{(Miller symbol)} \\
 3 P \tilde{3/2}^* = a : \frac{3}{2} b : 1c & \text{(parameters)} \\
 1 \quad \frac{2}{3} \quad \frac{1}{3} & \text{(indices)} \\
 3 \quad 2 \quad 1 & \text{(Miller symbol)} \\
 \text{and } 3 \ 6 \ 4 = 3a : 6b : 4c & \text{(indices)} \\
 \frac{1}{3} a : \frac{1}{6} b : \frac{1}{4} c & \text{(parameters)} \\
 2a : b : \frac{3}{2} c & \text{(parameter on } b = 1) \\
 \frac{3}{2} P \tilde{2}^* & \text{(Naumann symbol)} \\
 o \ 1 \ o = oa : b : oc & \text{(indices)} \\
 \infty a : b : \infty c & \text{(parameters)} \\
 \infty P \infty^* & \text{(Naumann symbol)}
 \end{array}$$

\* In the Naumann symbols the signs  $\sim$  and  $\vee$  refer to the  $a$  axis and the sign  $\sim$  to the  $b$  axis. When no sign is used above the parameter after the initial letter the parameter refers to the  $a$  and the  $b$  axes indifferently.

## CHAPTER IV.

### SYMMETRY.

**Symmetry.**—By the symmetry of a body is meant the *regular arrangement of its parts with respect to planes, lines, or points*. Four types of symmetry are recognizable in crystals, of which three are much more important than the fourth. These three are:

1. Symmetry with reference to planes.
2. Symmetry with respect to lines.
3. Symmetry with respect to points.

**Symmetry with Respect to Planes.**—When two halves of a body bear to each other the relation of an object and its image, the two halves are said to be symmetrical about the plane that divides them. When the body possesses only one such plane of symmetry, it is described as being symmetrical about a single plane. (See Fig. 16.) When it possesses three such planes, it is symmetrical about three planes. The planes are known as *planes of symmetry*.

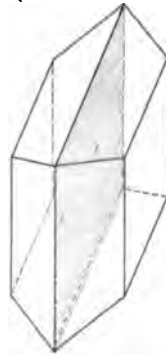


FIG. 16.—Body divided by one plane of symmetry.

**Symmetry with Respect to Lines.**—Many bodies may be revolved about a line so situated within it that a revolution through an angle less than  $360^\circ$  will cause its parts to occupy successively the same relative positions as they originally occupied. When such a condition is brought about by a revolution through  $180^\circ$ , or twice during a complete revolution, the body is said to possess a twofold or binary symmetry. When brought about by revolutions of  $120^\circ$ ,  $90^\circ$ , or  $60^\circ$ , it is spoken of as possessing threefold or ternary, fourfold or quadratic, and sixfold or hexagonal symmetry. The line about which such a revolution is possible is called an *axis of symmetry*.

The axes of twofold symmetry are designated as secondary

axes of symmetry; those of more than twofold symmetry are designated as principal axes of symmetry.

Symmetry axes are polar when their opposite ends terminate differently; i.e., in crystals, when their opposite ends terminate in different planes, edges, or solid angles (compare Fig. 33).

**Symmetry with Reference to Points.**—Bodies are symmetrical about a point when a straight line drawn through this point terminates at equal distances on its opposite sides in similar surfaces or angles. Such a point is called a *center of symmetry*. A crystal possessing a center of symmetry must be bounded by pairs of parallel planes (see Figs. 200 and 201).

A sphere and a cube are symmetrical about a point at their centers. A right cone with a circular base and a right pyramid with a square base are symmetrical about lines passing through their apices and the centers of their bases. The latter possesses a fourfold symmetry and has four vertical planes of symmetry, two passing through the corners of the base and two through the centers of its sides. A right pyramid with a rectangular base that is not a square possesses two vertical planes of symmetry through the centers of the sides of its base. A right pyramid with an isosceles triangle as its base possesses a single plane of symmetry passing vertically through its base and bisecting the angle between its equal sides.

**Symmetry in Crystallography.**—It has already been remarked that the inclinations of the planes on a crystal are of much more importance than their distances from its center. Similarly the notion of symmetry in crystals applies to the directions of its planes rather than to the distances of these planes from the plane of symmetry.

Crystallographically the two figures on the next page (Fig. 17) are symmetrical about the plane AB passed through them perpendicular to the plane of the paper. The two parts of figure 17 A are symmetrical not only with respect to the inclinations of their bounding lines, but also with respect to the distances of these lines from the plane of symmetry. This polygon is ideal in its symmetry. Figure 17 B is symmetrical with respect to the inclination of the lines on opposite sides of the plane AB, but

not with respect to the distances of these lines from the plane. Every line on either side of the plane of symmetry corresponds to a line on the opposite side, and the corresponding lines on each side make equal angles with the corresponding adjacent lines. The angles  $a, b, c$  are equal, respectively, to  $a', b', c'$ .

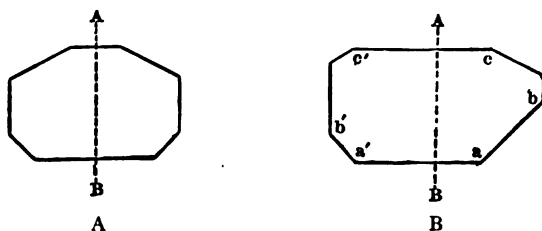


FIG. 17.—Diagrams illustrating symmetry.

In the same way the crystals B and C in figure 18 are symmetrical in the same degree as is the crystal A. But the latter crystal is ideal in its symmetry, while the other two are distorted through irregularities in their growth.

**Crystal Forms.**—One definition of a crystal form has been

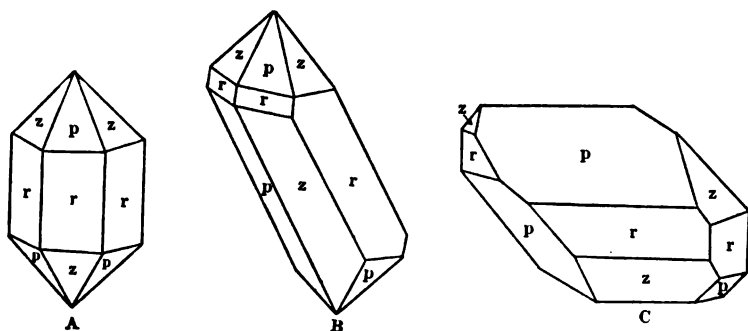


FIG. 18.—Three crystals bounded by the same planes, but with different habits.

given (page 21). It may better be defined as the sum of all the planes demanded by a crystal's symmetry, in consequence of the presence of one. It is for this reason that the symbol of a single plane of a form may be made to stand for the entire form.

Occasionally through accidents of growth one or more planes of a form may be crowded from the crystal. Plane  $z$  in the upper

left-hand corner of crystal C, figure 18, for instance, is very small. Continued growth of P and r might have caused it to disappear completely. An accident of this kind is not recognized in the symbol of a crystal, though the absence of the plane is usually mentioned in the description of the crystal's habit.

**Grades of Symmetry.**—Differences in the number of planes, axes, and centers of symmetry possessed by crystal forms determine their grade of symmetry—the greater the number of elements of symmetry present, the higher the grade of symmetry.

**Law of Symmetry.**—Each crystallizing substance possesses a characteristic grade of symmetry. *Only those forms may occur on its crystals that possess the same grade of symmetry.*

**Crystallographic Systems.**—All crystal forms possessing the same grade of symmetry and all forms that may be regarded as derived from these by the suppression of a certain definite proportion of their planes in accordance with certain definite laws (partial forms) are grouped together as a system.

**Two Kinds of Planes of Symmetry.**—Thus far we have discussed planes of symmetry without distinguishing between the different kinds. As a matter of fact, two kinds must be distinguished—principal and secondary planes.

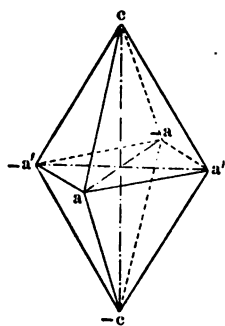


FIG. 19.—Crystal possessing principal and secondary planes of symmetry.

Principal planes of symmetry contain two or more equivalent and interchangeable directions—directions that may be imagined as interchanged, without affecting the shape of the crystal through which they pass. They are symmetry planes to which two or more other symmetry planes are perpendicular.

Secondary planes of symmetry possess no interchangeable directions.

In the form represented by figure 19, the plane  $a, a', -a, -a'$ , is a principal plane of symmetry, while the plane  $a, c, -a, -c$  is a secondary plane. The directions  $a -a$ , and  $a' -a'$  may be interchanged by a horizontal revolution of the crystal through an arc of  $90^\circ$  without affecting the shape of the crystal in the least. On the other hand, the directions repre-

sented by the lines  $a - a$  and  $c - c$  are not equivalent directions, because if the crystal were revolved about  $a' - a'$  through an arc of  $90^\circ$  until  $c - c'$  takes the direction now held by  $a - a$ , its long axis would be horizontal, whereas originally it was vertical.\*

**Number and Characterization of Crystal Systems.**—It can be proven mathematically that all the crystal forms that are possible may be divided into 32 groups or classes, each characterized by a special grade of symmetry. To each class a distinctive name is given which indicates its grade of symmetry. It can also be shown that of these groups there are six from which the other 26 may be considered as derived. There are, therefore, six systems into which all crystals may be grouped. Each system includes, therefore, not only the forms possessing a certain grade of symmetry, but also all forms that may be considered as derived from these; i.e., all forms that may be referred to the same system of coordinates or axes. The groups or systems generally recognized are as follows:

<i>System</i>	<i>Forms</i>
Isometric.....	With 3 principal and 6 secondary planes of symmetry, and partial forms derived from these.
Hexagonal.....	With 1 principal and 6 secondary planes of symmetry, and partial forms derived from these.
Tetragonal.....	With 1 principal and 4 secondary planes of symmetry, and partial forms derived from these.
Orthorhombic....	With 0 principal and 3 secondary planes of symmetry, and partial forms derived from these.

\* It is absolutely necessary that the student should understand the distinction between principal and secondary planes of symmetry before proceeding further. An excellent exercise for this purpose is to separate a number of crystal models, such as are recommended in the footnote, page 11, into groups in accordance with the crystal system represented by them.

**Monoclinic . . . . .** With 0 principal and 1 secondary plane of symmetry, and partial forms derived from these.

**Triclinic . . . . .** With no plane of symmetry, but only a center of symmetry, and partial forms derived from these.

## CHAPTER V.

### THE ISOMETRIC OR REGULAR SYSTEM.

**Division of the System.**—The forms belonging to the isometric system like those belonging to all the other systems may be separated into two groups, viz., holohedral and partial forms.

The holohedral (ὅλος, whole, and ἔδρα, face) forms are those possessing all the planes demanded by the complete symmetry of the system. A form of this kind is known as a *holohedron*. The partial forms are made up of the same planes as the corresponding holohedral forms, but some of those demanded by the complete symmetry of the system are not present. Partial forms are of a lower grade of symmetry than are the holohedrons. In some cases only half of the whole number of planes found in the holohedron occur on the partial form. This condition is known in some instances as *hemihedrism*, in others as *hemimorphism*. In other cases only  $1/4$  of the planes persist. Such a condition is known as *tetartohedrism*.

### HOLOHEDRAL DIVISION.

(*Hexoctahedral Class.*)

**Complete Symmetry of the System.**—The holohedral forms of the isometric system possess three principal planes of symmetry at right angles to one another and six secondary planes which bisect the right angles between the principal planes (Fig. 20). In addition to these 9 planes of symmetry the holohedrons of this system possess also 3 principal axes of fourfold symmetry that are perpendicular to the three principal planes of symmetry, 4 axes of threefold symmetry passing through the centers of the octants included between the three principal planes of symmetry, 6 similar axes of twofold or binary symmetry perpendicular to the secondary planes of symmetry, and one center of symmetry.



The presence of the center of symmetry demands the presence on the forms of pairs of parallel planes on opposite sides of the center of symmetry, which must be at the geometrical center of the form.

Figure 21 represents in a schematic way the elements of symmetry in the holohedrons of this system. In this and in corresponding figures on later pages the characters of the axes of symmetry are represented by symbols at their terminations as follows: ■ fourfold or tetragonal symmetry; ▲ threefold

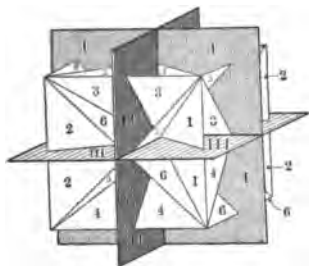


FIG. 20.—The planes of symmetry of the isometric holohedrons.

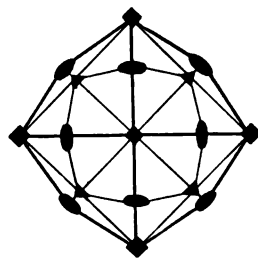


FIG. 21.—Elements of symmetry in the isometric holohedrons.

or trigonal symmetry; ● twofold or binary symmetry. Hexagonal symmetry is represented by ●. The positions of the principal planes of symmetry are indicated by heavy lines, and those of secondary planes by lighter lines.

**Axes.**—The lines chosen as the axes in this system are the *intersections of the three principal planes of symmetry*. From the relative positions of these planes with respect to one another the axes *must necessarily be three lines perpendicular to each other at a common point*. Since, moreover, a secondary plane of symmetry bisects each of the angles between the axes, *all these axes must be equivalent*; i.e., the axes must be equal in length (or the distances taken as unity on each must be equal) and they must be equally terminated by crystal planes (i.e., by the same number of similar crystal planes).

The axes of the isometric system may therefore be defined

as three lines of equal unit lengths perpendicular to each other at a common point (Fig. 13, page 17). Since the axes are imaginary lines drawn within a crystal, they extend indefinitely.

It must be borne in mind that whenever length is referred to in discussions of crystals *relative*, and *not absolute*, length is meant. From the term "equal unit lengths" we must not infer that the unit length in all crystals of the isometric system is the same absolute length, but rather that the ratio between the unities on the axes of all isometric crystals is always one.

**The Groundform of the System.**—The groundform of a system has been defined as that form made up of planes cutting the axes at unity distances. In the isometric system the planes of the groundform cut the three axes at the same distance from the center; i.e., the ratio between their intercepts is 1 : 1 : 1. When one plane of this form is present, symmetry demands the presence of seven others, so that the entire form consists of eight planes, which together comprise an *octahedron* (Fig. 22). Its interfacial angles are each about  $109^{\circ} 28' 1/4''$ .

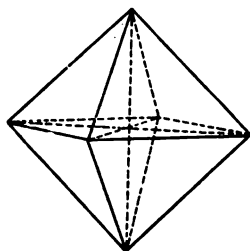


FIG. 22.—Groundform of the isometric system.

The parameters on the three axes are unity. The symbol of a plane is  $a : b : c$ , with the proper signs appended, or, since the axes are equivalent,  $a : a : a$ , and the symbol of the form is O. The indices are 1 1 1.

**The Most General Form in the System.**—The most general form in the system is that composed of planes cutting the three axes at different distances, no one of which is zero or infinity. Since one of these distances may be considered as unity, the symbol of one of its planes is  $na : b : mc$ . Since, moreover, the different axes are equivalent or since the symmetry of the system requires that each axis shall be terminated in the same manner, it follows that the presence of the plane  $na : b : mc$  necessitates the presence of five other planes in the same octant, for without introducing any other than the plus signs understood in the above symbol, this symbol may be changed five times by

using the same parameters before each of the three axes successively, as:

$$\begin{array}{lll} na : b : mc & na : mb : c & a : mb : nc \\ a : nb : mc & ma : nb : c & ma : b : nc \end{array}$$

There are thus in this form six planes in an octant. As there are eight octants in all, the total number of planes in the form must be 48.

We can now understand why in this system we may write the symbol of the axes  $a : a : a$  instead of  $a : b : c$ . Each axis is affected in the same manner by similar planes. There is no character by which one axis is distinguished from the others, and hence we may indicate them all by the same letter. The symbol of the individual planes is  $na : a : ma$ , with the proper signs, and that of the form  $n O m$  or  $m O n$ . Its indices are  $h k l$ , in which the three different letters stand for three different values.

**The Hexoctahedron.**—The most general form of the system is known as the *hexoctahedron* (Fig. 23), because it possesses six faces in an octant; or it is a form that may be regarded as an octahedron in which an octahedral face is replaced by six equal faces, each of which is a triangle. Each face cuts one axis at a distance taken as unity, another at  $n$  times this distance, and the third at  $m$  times this distance. The interfacial angles are of three kinds (o, c, and d in Fig. 23). Their values vary with the values of  $m$  and  $n$ , the figure

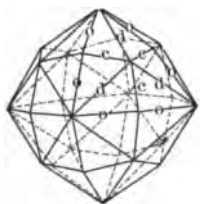


FIG. 23.—Hexoctahedron,  $mOn$ , or  $hkl$ .

approaching more and more nearly the habit of the octahedron as  $m$  and  $n$  approach unity in value. The symbols  $2 O 3$  and  $3 O 4$  both represent hexoctahedra. They signify forms made up of planes cutting the three axes at distances whose ratios are  $2 : 1 : 3$  and  $3 : 1 : 4$ . Their indices are  $3 \ 6 \ 2$  and  $4 \ 12 \ 3$ .

**Other Forms Derived from the General Form.**—In the symbol for the general form  $na : a : ma$ , the parameters  $n$  and  $m$  may be made to vary between 0 and  $\infty$ . If we give  $n$  and  $m$  all the different values possible, remembering that a plane which does not meet the axis cuts it at  $\infty$ , we obtain the following symbols:

$ma : a : ma$ , when  $n = m$ ; i.e., when both parameters are equal, and neither is unity nor infinity.

$a : a : ma$ , or  $na : a : a$ , when  $m$  or  $n = 1$ ; i.e., when one of these parameters becomes unity.

$\infty a : a : ma$ , or  $na : a : \infty a$ , when  $m$  or  $n = \infty$ ; i.e., when one of these parameters becomes infinity.

$a : a : a$ , when  $m$  and  $n = 1$ ; i.e., when both parameters become unity.

$\infty a : a : \infty a$ , when  $m$  and  $n = \infty$ ; i.e., when both parameters become infinity.

$a : a : \infty a$ , or  $\infty a : a : a$ , when  $m$  or  $n = 1$ , and the remaining  $n$  or  $m$  equals infinity.

Thus there are only seven possible kinds of forms in the regular system, for  $a : a : ma$  and  $na : a : a$  are the same kind of form, as are also  $\infty a : a : ma$  and  $na : a : \infty a$ , and  $a : a : \infty a$  and  $\infty a : a : a$ .

The last two symbols are symbols of different planes on the same form, while  $\infty a : a : ma$  and  $na : a : \infty a$  are forms in which the parameter which is neither unity nor infinity is some other value which is different in the two forms. One may be  $2 \text{ O } \infty$  and the other  $3 \text{ O } \infty$ , on each of which the distribution of the planes is the same although their interfacial angles are different.

**Forms Composed of 24 Planes.**—Of the forms indicated above three possess 24 faces each. These are those made up of planes with the symbols  $ma : a : ma$ ,  $a : a : ma$ , and  $\infty a : a : ma$  respectively. The symbols of the forms are  $m \text{ O } m$ ,  $m \text{ O}$ , and  $\infty \text{ O } m$ . The values of their interfacial angles depend upon the value of  $m$ . The corresponding indices are  $hll$ ,  $hlh$  and  $hlo$  in which  $h > l$ .

*The Icositetrahedron.*—The first of these is the icositetrahedron (Fig. 24) or the 24-sided figure, consisting of a polyhedron bounded by 24 similar trapeziums, three in each octant. Each plane cuts one axis at unity and the other two axes at some distance greater than unity and less than infinity. The interfacial angles are two in kind ( $o$  and  $c$  of Fig. 23). Examples  $2 \text{ O } 2$  and  $3 \text{ O } 3$ , or  $211$  and  $311$ .

*The Trisoctahedron.*—The second form, known as the trisoc-

tahedron (Fig. 25) is bounded by three isosceles triangles in each octant, so distributed that their apices meet at a common point, which is equally distant from the terminations of all the axes. Each plane cuts two axes at unity and one at  $m$  and each makes with the contiguous planes two different interfacial angles  $o$  and  $d$  (of Fig. 23). Examples 2 O and 3 O, or 212 and 313.

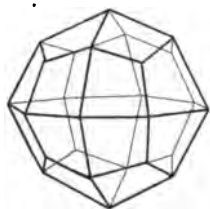


FIG. 24.—Icositetrahedron,  $m O m$  or  $hll$ .

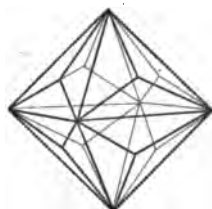


FIG. 25.—Trisoctahedron,  $m O$ , or  $hll$ .

*The Tetrahexahedron.*—The remaining form is also bounded by 24 isosceles triangles (Fig. 26), but they are in groups of four with their apices at a common point which is the termination of an axis. Each plane meets one axis at unity, another at some distance greater than unity, and is parallel to the third. Two different interfacial angles are present. They correspond to the  $c$  and  $d$  edges of the hexoctahedron. Examples  $\infty O 2$ ,  $\infty O 4$ , 3 O  $\infty$ , or 210, 410, and 310.

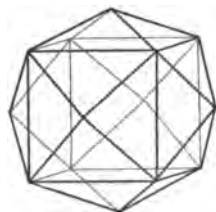


FIG. 26.—Tetrahexahedron,  $m O \infty$  or  $hll^0$ .

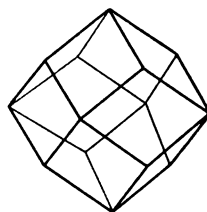


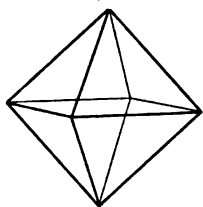
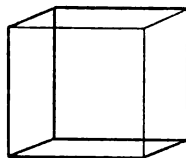
FIG. 27.—Dodecahedron,  $\infty O$  or  $110$ .

**Form Composed of 12 Planes.**—*The Dodecahedron* possesses 12 planes (Fig. 27), each of which is at the same time in two octants. All are similar rhombs, four of which meet at each termination of the axes, and each cuts two axes at the same distance and is parallel to the third. Their symbols are  $a : a : \infty a$ .

The interfacial angles are each  $120^\circ$ , and since there are no variable parameters possible in the form there can be only one dodecahedron in the system. The symbol of the form is  $\infty O$  and its indices 110.

**Form Composed of 8 Planes.**—*The Octahedron*,  $O$ , has already been discussed. It consists of eight equilateral triangular faces, each in an octant. Its interfacial angles measure  $109^\circ 28' 16''$ . (See Fig. 28.)

**Form Composed of 6 Planes.**—*The Cube*,  $\infty O \infty$ , contains the smallest number of planes possible to holohedrons in the isometric system (Fig. 29). Each of the planes is a square, one of which is perpendicular to each axis at its termination. The interfacial angles are all  $90^\circ$ . The indices are 100.

FIG. 28.—Octahedron,  $O$  or 111.FIG. 29.—Cube,  $\infty O \infty$  or 100.

**Summary.**—The total number of holohedral forms possible in the isometric system is seven. Of these, one possesses 48 planes, 3 possess 24 planes each, one possesses 12 planes, one, 8 planes, and one 6 planes. Of these three, the octahedron, the cube, and the dodecahedron, are forms that possess constant interfacial angles, consequently only one of each is possible. The symbols of the other four forms possess variable parameters, hence there may be as many of each of these forms in the system as there are values that may be assumed by their parameters. Their number is limited only by the law of the rationality of the indices.

**Determination of the Forms.**—On crystals the symbols that represent the different forms are determined from the interfacial angles made by their planes with each other. By measurement with the goniometer the values of the interfacial angles between selected planes are obtained. From these, by the methods of

spherical geometry, the value of the intercept on one axis may be calculated in terms of its intercept on another. If the latter be taken as the unity intercept the value determined is the parameter of the plane on the other axis.

If in the figure ( $\infty O m$ , Fig. 30) the angle  $C$  is measured, then by calculation we can determine the relative distances at which the plane  $t$  cuts the two axes  $Oa$  and  $Oc$ . If the distance at

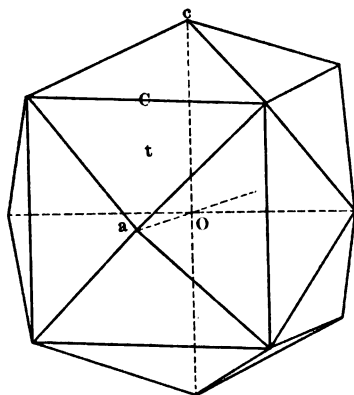


FIG. 30.

which the plane cuts  $Oa$  be taken as unity, then the intercept on  $Oc$  measured in terms of this unity is the parameter on  $Oc$ . If the value of the angle is  $126^\circ 52'$ , calculation will show that the plane cuts  $Oc$  at 3 times the distance at which it cuts  $Oa$ . Its symbol is  $a : \infty a : 3a$  or  $3 O \infty$ .

**Simple Forms and Combinations.**—Each of the figures reproduced above represents a simple form; i.e., a polyhedron all of whose planes are necessi-

tated by the presence of a single one. All the planes on each form may be represented by a single symbol.

In nature, while crystals occur that are bounded by the planes of a single form, it is more usual to find on them planes belonging to several forms. Such crystals must be represented by as many symbols as there are planes on them belonging to different forms. The total number of planes present is equal to the sum of all the planes belonging to all the forms represented by the symbols.

If a crystal contains planes belonging to the forms  $2 Q_3$ ,  $\infty O$  and  $\infty O \infty$ , then the total number of planes on the crystal must be  $48 + 12 + 6$  or 66.

The occurrence of two or more forms on a crystal is known as a *combination of forms*. The symbol of a combination consists of the symbols of its different forms written in order, with the symbol for the form with the largest faces first.

The forms occurring on the crystal represented in figure 31 are the cube (h) and the dodecahedron (d). Its symbol is  $\infty O \infty, \infty O$ . In figure 32 we have combinations of the cube, octahedron, and dodecahedron. On the crystal to the left O predominates, on that to the right  $\infty O \infty$  is the largest form. The symbols for the combinations are O,  $\infty O \infty$ ,  $\infty O$  and  $\infty O \infty$ , O,  $\infty O$ . When written in the proper order they indicate the general habit of the crystal which they represent. The first of these two symbols indicates that the crystal which it represents is octahedral in habit, and the second that the corresponding crystal possesses a cubical habit.

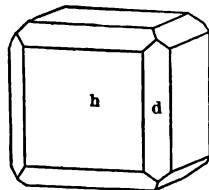


FIG. 31.—Combination of cube ( $\infty O \infty$ ) and dodecahedron ( $\infty O$ ).

NOTE.—In writing the symbols of combinations it is necessary first to fix the position of the axes. This is done by determining the positions of the principal planes of symmetry in the crystal (or model) and noting their lines of intersection. These three lines of intersection are the three axes to which all the planes must be referred. *When once fixed they remain the axes for all the planes in the combination.* If the choice of the axes is the correct one, *their six ends terminate at similar points on the crystal.*

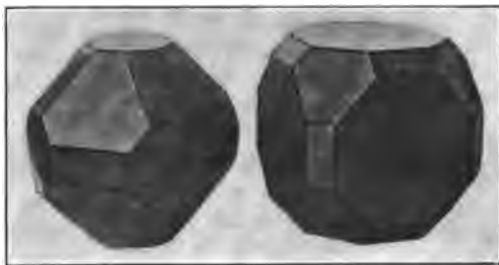


FIG. 32.—Combinations of octahedron, cube and dodecahedron.

The next step is to determine the relations of the several faces to these axes. A close inspection of the planes will often suffice to indicate the approximate relative distances at which they cut the axes. If this is not easily seen, a glass plate laid flat



against a plane will serve to aid in this determination. The distances from the center of the crystal at which the plate cuts the axes, or their prolongations, afford the means for estimating the lengths of the parameters of the plane on which the plate lies. Suppose, for instance, the glass plate has been laid on a plane whose symbol is sought. If the plate meets one axis at a certain distance, a second at some greater distance, and the third at a still greater distance, then, since the shortest of these distances may be made unity, the symbol of the plane is  $a : ma : na$  or the form to which it belongs is  $m O n$ . Now, suppose the plate be laid on a second plane. It may cut one axis at a certain distance and the other two at similar distances which, however, are different from the distance at which it cuts the first axis. The symbol of the plane is either  $a : ma : ma$ , or  $ma : a : a$ . The shortest distance is always made unity. Consequently, if the two axes that are cut at the same distance are cut nearer the center of the crystal than is the other axis, the symbol of the plane is  $ma : a : a$ , which is a plane of form  $mO$ . If, on the other hand, these two axes are cut at a greater distance from the center than the third axis, the plane is  $a : ma : ma$ , or a plane of the form  $m O m$ .

## CHAPTER VI.

### PARTIAL FORMS—HEMIHEDRISM AND TETARTOEDRISM OF THE ISOMETRIC SYSTEM.

**Independent Occurrence of Partial Crystal Forms.**—The holohedral forms that have been discussed are those composed of the full number of planes which the complete symmetry of the isometric system demands. Sometimes, in consequence of irregular growths, certain planes on a crystal may be developed at the expense of other planes, until finally in extreme cases the latter may be reduced to mere points, and so disappear as planes. This irregular disappearance of some planes of a complete form is accidental, and is, consequently, of no crystallographic significance. The condition is known as *merohedrism*, and the incomplete form is said to be *merohedral*.

In the crystal of quartz ( $\text{SiO}_2$ ) represented in figure 11 the growth was not symmetrical about the axis, and so the crystal is distorted. The plane *z* at the upper left-hand corner has been crowded by the abnormal development of the planes *P* and *r*, until it has nearly disappeared. If the crystal had continued its growth in the same direction, *z* would probably have disappeared, and the form to which it belongs would have been represented by one less plane than normally belongs to it.

**Hemimorphism.**—Occasionally all the planes of half the holohedral form occur at one termination of an axis of symmetry, while from the other end of the same axis they are absent, or are represented by all the planes belonging to half of some other form; i.e., the symmetry axis is polar. It is to be noted that in cases like this all the planes of the given holohedral form that are naturally expected to be present around one termination of an axis are present while none are present at the other end of the same axis. This condition is known as *hemimorphism*. It is characteristic of a few minerals which always exhibit it and hence must be the direct result of the internal arrangement of their molecules.

That this is the case is shown by the physical properties of hemimorphic crystals. When these are heated they become pyroelectric, one end becoming charged with positive electricity and the other end with negative electricity. The former is known as the analogue pole, and the latter as the antilogue pole. In writing the symbols of hemimorphic crystals the pole about which the several hemimorphic forms occur must be indicated. (See also pages 216-217.)

Figure 33 represents a hemimorphic crystal of calamine ( $\text{Zn}(\text{OH})_2\text{SiO}_3$ ), an orthorhombic mineral. The two ends of the  $c$  axis are differently terminated, the lower end being terminated by one-half the planes of one form and the upper end by one-half the planes belonging to two other forms.

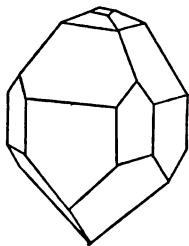


FIG. 33.—Crystal exhibiting hemimorphism.

#### **Hemihedrisms and Tetartohedrisms.**—

In addition to the merohedral and hemimorphic forms there are other partial forms observed on crystals that are of great importance, since their presence characterizes a large number of substances. These forms do not possess the symmetry of the holohedrons belonging to any system, but they are bounded by planes that are the same geometrically as those on certain holohedrons from which they may be regarded as being derived. When the planes that persist in the partial form are one-half of those present on the corresponding holohedron, the new form is called a *hemihedron*, or it is said to be *hemihedral*. When only one-quarter of the planes persist, the new form is *tetartohedral*.

**Law of Hemihedrisms and Tetartohedrisms.**—If the only condition of hemihedrisms and tetartohedrisms were the occurrence of one-half or one-quarter of the planes of the holohedron, the number of new hemihedrons and tetartohedrons that might exist would be very large. Experience shows, however, that only those planes persist in the partial forms which terminate equivalent ends of the crystallographic axes in a similar manner. In other words, in hemihedrons and tetartohedrons which are not hemimorphic,

*equivalent terminations of the crystallographic axes are equivalently terminated.* That is: All equivalent terminations of the axes in hemihedrons and tetartohedrons (terminations that are separated by planes of symmetry in the holohedrons) must be cut by the same number of similar planes. This limitation of the planes that occur in the new forms reduces their number to comparatively few.

We may imagine any hemihedral or tetartohedral form as being produced by the suppression of a certain half or three-quarters of the planes composing the complete or holohedral form and the extension of the remaining planes until they meet. Suppose, for instance, that the white planes of the inside figure (Fig. 34) are suppressed and that the shaded planes are extended to intersection: they will then produce the outside figure, which is the hemihedron corresponding to the interior holohedron. In this crystal the terminations lettered A, B, C, D are all equivalent in the holohedral form, consequently in the hemihedral form they must be cut by the same number of equivalent planes. An inspection of the figure will show that each is cut by two planes which are equally inclined to each axis. The axis from E to F is not equivalent to the axes terminating at A, B, C, D, hence its extremities are not terminated in the same way as are A, B, C, D. But there is a plane of symmetry in the holohedral form perpendicular to this axis, hence its two terminations are equivalent, and consequently in the hemihedron they should be similarly terminated. As a matter of fact, E and F terminate in three planes each, and the planes that cut at E are inclined to the axis at exactly the same angles as those at which the three planes cutting at F are inclined to it.

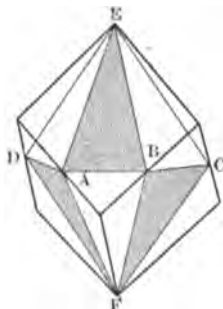


FIG. 34.—Crystal exhibiting hemihedrism. Outside form contains one-half the planes of the enclosed form.

#### **Combinations of Hemihedral and Tetartohedral Forms.—**

As in the case of holohedral forms, combinations of hemihedral and of tetartohedral forms are frequently met with in nature.

Since, however, the external forms of crystals are but the expressions of definite plans of internal structure, it must necessarily follow that combinations of forms are limited to those of the same grade of symmetry. (See statement of Law of Symmetry, page 28.) Holohedral forms combine with holohedral forms; hemihedral with hemihedral, and tetartohedral with tetartohedral. Further, only those hemihedral forms possessing the same symmetry may combine with one another, and only those tetartohedrons with the same symmetry.

Some holohedral forms, when subjected to the suppression of one-half or of three-quarters of their planes yield by the extension of the remaining planes new forms that are geometrically identical with the originals; i.e., they are hemihedrons that are identical geometrically with holohedrons. Such apparently holohedral forms may combine with hemihedral or with tetartohedral forms that may be considered as having been derived from other holohedral forms by the same method as was applied to the forms that appear to be holohedrons, and thus there is seemingly an exception to the general statement made above. But this supposed exception is not a real one, since the apparently holohedral forms are found to possess all the properties of hemihedral ones, except that of shape.

#### **HEMIHEDRISM OF THE ISOMETRIC SYSTEM.**

**Number of New Hemihedral Forms in the Isometric System.**—The whole number of new hemihedral forms that are directly related to the holohedrons of the isometric system is seven. The other hemihedral forms in this system are geometrically identical with holohedrons.

**Grouping of the Hemihedrons.**—The seven new hemihedrons of the isometric system may be grouped into three classes in accordance with their grade of symmetry. They are all of a lower grade of symmetry than the holohedrons of the system, as would be expected from the fact that they possess fewer faces than those demanded by the condition of holohedrism. Some of them have lost all the principal planes of symmetry that are found in the holohedrons, others have lost all the secondary

planes of symmetry, and another has lost all of its symmetry planes of both kinds.

The forms of each of the three groups may be considered as being derived from holohedrons by suppressing a certain half of the holohedral planes and allowing the other half to extend until they intersect. In order, however, that the persisting planes may comply with the condition of hemihedrism, which demands that in the hemihedron equivalent terminations of the crystallographic axes must be equivalently terminated, the derivation of the hemihedrons from holohedrons must take place in one of three ways, each of which gives rise to one of the groups already



FIG. 35.



FIG. 36.



FIG. 37.

Figures illustrating the three possible methods of derivation of the hemihedrons in the isometric system. Fig. 35, gyroidal, Fig. 36, pentagonal, and Fig. 37 tetrahedral.

referred to. The only three possible ways by which the desired result may be accomplished are illustrated in the three figures (Figs. 35, 36, 37).

The first figure represents the manner in which hemihedrons without planes of symmetry may be derived; i.e., by the suppression of alternate planes on the hexoctahedron. Figure 36 represents the way in which hemihedrons retaining the principal planes of symmetry may be derived; i.e., by the suppression of pairs of hexoctahedral planes that intersect in the principal planes of symmetry, or those holohedral planes that lie in the alternate sections of the 12 included within the secondary planes of symmetry occurring in the holohedrons. Figure 37 represents the way in which the third group of hemihedrons may be derived; i.e., by the suppression of all the planes in alternate octants.

If we imagine either the white or the shaded planes to disap-

pear and the others to be extended until they intersect, three new forms will result, each of which will satisfy the conditions of hemihedrism. No other method of suppressing half the planes on this form will result in new forms complying with these conditions. *Thus three distinct kinds of hemihedrism are possible in this system, and only three.* The new forms are known as *gyroidal, parallel, or pentagonal, and inclined or tetrahedral.*

**Gyroidal Hemihedrism** (*Pentagonal Icositetrahedral Class*).—Although gyroidal hemihedral forms are known on crystals, they are rare and consequently they will not be discussed. One of the

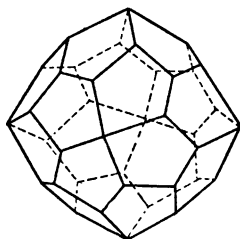


FIG. 38.—Right pentagonal icositetrahedron,  $r \frac{mOn}{2}$ .

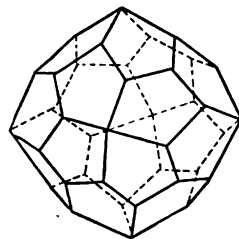


FIG. 39.—Left pentagonal icositetrahedron,  $l \frac{mOn}{2}$ .

two gyroidal hemihedrons derived from  $mOn$  is represented in figure 38. This is known as the right form and has the symbol  $r \frac{mOn}{2}$ , or  $r(hkl)$ . The left form,  $l \frac{mOn}{2}$ , or  $l(hkl)$ , is represented in figure 39.

**Pentagonal Hemihedrism** (*Dyakisdodecahedral Class*).—The different types of pentagonal hemihedrons are two in number. One is derived from the holohedron  $mOn$  and the other from  $mO\infty$ . Both retain the three principal planes of symmetry, the four trigonal axes of symmetry and the center of symmetry. The three axes of fourfold symmetry become axes of binary symmetry. These elements are indicated in figure 40, which indicates the character of the symmetry that would result if all the shaded planes of  $mOn$  or all the white ones should disappear. In other words, it represents the symmetry of all the white planes alone or of all the shaded ones. The heavy

solid lines indicate the positions of the planes of symmetry that would remain, and dotted lines the positions of those that would drop out.

*The Diploids.*—By the application of the pentagonal method of selection to the planes of the hexoctahedron, two hemihedral forms are produced that differ according to the positions of the original planes that survive. (Figs. 41 and 42.) Both are known as diploids. They are identical in shape, but differ in their positions with respect to the axes. Either one, by the revolution of  $90^\circ$  about either of its crystallographic axes, may be brought exactly into the position of the other. Forms that bear this relation to each other are said to be *congruent*.

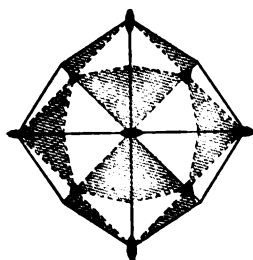


FIG. 40.—Diagram illustrating the distribution of the elements of symmetry in pentagonal hemihedrons.

The one outlined in heavy lines in figure 41 is designated as the positive *dyakisidodecahedron*, or *diploid*, and the other (Fig. 42) as the negative one. Their symbols are  $+\left[\frac{mOn}{2}\right]$  and  $-\left[\frac{mOn}{2}\right]$ . Their indices are  $\pi(hkl)$  and  $\pi(hlk)$ . Each form is bounded by



FIG. 41.—Positive diploid or dyakisidodecahedron,  $+\left[\frac{mOn}{2}\right]$  or  $\pi(hkl)$ .

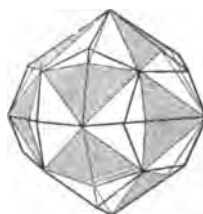


FIG. 42.—Negative diploid or dyakisidodecahedron,  $-\left[\frac{mOn}{2}\right]$  or  $\pi(hlk)$ .

twenty-four similar trapeziums meeting in three kinds of interfacial angles. There are, of course, as many pairs of diploids possible as there are hexoctahedra. Their shapes will necessarily vary with the values of  $m$  and  $n$ .

*The Pentagonal Dodecahedrons or the Pyritohedrons.*—The only other hemihedron of this class may be derived from the



tetrahexahedron. The extension of alternate planes on the holohedron corresponds exactly to the extension of alternate pairs of hexoctahedral planes that intersect in the planes of symmetry.

The new forms derived by the extension of alternate planes of the tetrahexahedron are the pyritohedrons (Figs. 43 and 44).



FIG. 43.—Positive pyritohedron,  $+\left[\begin{smallmatrix} mO^\infty \\ 2 \end{smallmatrix}\right]$  or  $\pi(hlo)$ .

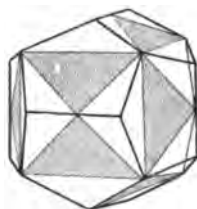


FIG. 44.—Negative pyritohedron,  $-\left[\begin{smallmatrix} mO^\infty \\ 2 \end{smallmatrix}\right]$  or  $\pi(hol)$ .

They are congruent forms bounded by twelve similar pentagons meeting in two different interfacial edges. Their symbols are  $+\left[\begin{smallmatrix} mO^\infty \\ 2 \end{smallmatrix}\right]$  (Fig. 43) and  $-\left[\begin{smallmatrix} mO^\infty \\ 2 \end{smallmatrix}\right]$  (Fig. 44). Their indices are  $\pi(hlo)$  and  $\pi(hol)$ .

**Tetrahedral Hemihedrism** (*Hextetrahedral Class*).—The extension of all the planes occurring in alternate octants of

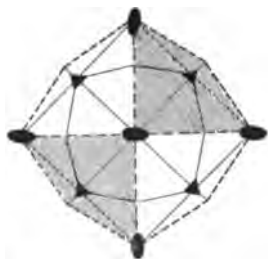


FIG. 45.—Diagram illustrating the distribution of the elements of symmetry in tetrahedral hemihedrons.

holohedral forms in the isometric system and the suppression of those in the other octants will produce new forms in all cases except the tetrahexahedron, the dodecahedron, and the cube. These three holohedrons yield hemihedral forms that are geometrically indistinguishable from themselves. Of the four new geometrical forms thus produced all have a tetrahedral habit, hence the name of the class. The

forms are congruent, each holohedron yielding two hemihedrons, which are distinguished as the positive and negative forms.

In the tetrahedral hemihedrons the principal planes of sym-

metry, the six axes of binary symmetry, and the center of symmetry have disappeared. The three axes of fourfold symmetry have become axes of binary symmetry. There remain the six secondary planes of symmetry and the four trigonal axes of sym-



FIG. 46.—Positive hextetrahedron,  $+\frac{mOn}{2}$  or  $\kappa(hkl)$ .



FIG. 47.—Negative hextetrahedron,  $-\frac{mOn}{2}$  or  $\kappa(hkl)$ .

metry. These axes are now, however, polar. The forms contain no parallel planes (Fig. 45).

*The Hextetrahedrons.*—The hextetrahedrons are 24-sided figures bounded by scalene triangles meeting in 3 kinds of edges.

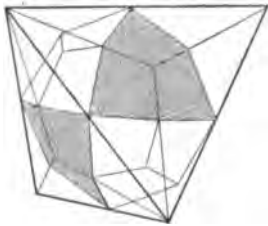


FIG. 48.—Positive tristetrahedron,  $+\frac{mOn}{2}$  or  $\kappa(hll)$ .

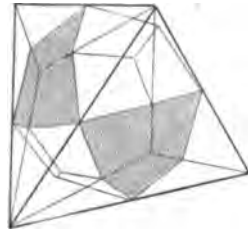


FIG. 49.—Negative tristetrahedron,  $-\frac{mOn}{2}$  or  $\kappa(hll)$ .

The crystal axes terminate in the solid angles at which 4 planes meet. Figure 46 is the positive form,  $+\frac{mOn}{2}$ , and figure 47, the negative form,  $-\frac{mOn}{2}$ . Their indices are  $\kappa(hkl)$  and  $\kappa(hkl)$ .

*The Trigonal Tristetrahedrons.*—The trigonal tristetrahedrons are composed of half the planes of the icositetrahedron. Each is

bounded by 12 similar isosceles triangles intersecting in two kinds of edges. The crystal axes terminate in the centers of the long edges. Figure 48 represents the positive form,  $+\frac{mOm}{2}$ , and figure 49, the negative one,  $-\frac{mOm}{2}$ . Their corresponding indices are  $\kappa(hll)$  and  $\kappa(h\bar{l}l)$ .

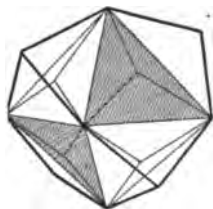


FIG. 50.—Positive deltoïd dodecahedron,  $+\frac{mO}{2}$  or  $\kappa(hlh)$

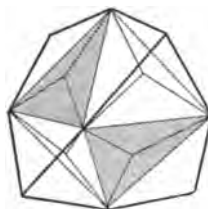


FIG. 51.—Negative deltoïd dodecahedron,  $-\frac{mO}{2}$  or  $\kappa(h\bar{l}h)$ .

*The Tetragonal Tristetrahedrons or Deltoïd Dodecahedrons.*—There are two of these derived from the trisoctahedron as indicated in figures 50 and 51. They are bounded by 12 trapeziums intersecting in two kinds of interfacial angles. The crystallo-

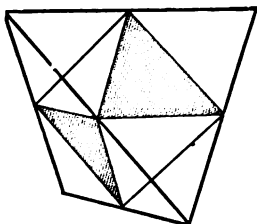


FIG. 52.—Positive tetrahedron  $+\frac{O}{2}$  or  $\kappa(III)$ .

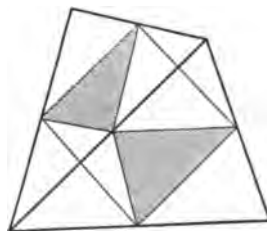


FIG. 53.—Negative tetrahedron,  $-\frac{O}{2}$  or  $\kappa(I\bar{I}I)$ .

graphic axes terminate in the solid angles at which four planes meet. Their symbols are  $+\frac{mO}{2}$  (Fig. 50) and  $-\frac{mO}{2}$  (Fig. 51), and their indices are  $\kappa(hlh)$  and  $\kappa(h\bar{l}h)$ .

*The Tetrahedrons.*—These forms are derived from the octahedron. There are two of them, each bounded by four equi-

lateral triangles intersecting in six similar interfacial edges (Figs. 52 and 53). The crystallographic axes terminate in the centers of these edges. Their symbols are  $+\frac{O}{2}$  and  $-\frac{O}{2}$ . Their indices are  $\kappa(\text{III})$  and  $\kappa(\text{III})$ .

NOTE.—The hemihedrons derived from  $mOm$  possess triangular faces, while the holohedrons are bounded by trapeziums. On the other hand, the hemihedron derived from  $mO$  is composed of trapeziums, while the faces of the holohedron are triangles.

#### TABULAR SUMMARY OF ISOMETRIC HEMIHEDRONS.

*(The forms with symbols not written as fractions are indistinguishable from the holohedrons by their shapes, i.e., they are geometrically identical with the corresponding holohedrons.)*

Holohedrons	Hemihedrons		
	Gyroidal	Pentagonal	Tetrahedral
$mOn$	$mOn$ 2 r.l.	$\pm \begin{bmatrix} mOn \\ 2 \end{bmatrix}$	$\pm \frac{mOn}{2}$
$mO\infty$	$mO\infty$	$\pm \begin{bmatrix} mO\infty \\ 2 \end{bmatrix}$	$mO\infty$
$mOm$	$mOm$	$mOm$	$\pm \frac{mOm}{2}$
$mO$	$mO$	$mO$	$\pm \frac{mO}{2}$
$\infty O$	$\infty O$	$\infty O$	$\infty O$
$\infty O\infty$	$\infty O\infty$	$\infty O\infty$	$\infty O\infty$
$O$	$O$	$O$	$\pm \frac{O}{2}$

**Combinations of Hemihedrons.**—Hemihedrons combine with each other exactly as do holohedrons. They may combine with other hemihedrons of the same grade of symmetry, but not with those of different grades of symmetry. The combining

forms may all be hemihedrons with distinctive forms different from holohedral forms, or they may be in part hemihedrons that are geometrically identical with holohedrons, provided the latter are not such as may yield new hemihedrons with the grade of symmetry of the combining forms. In other words, the hemihedrons in each vertical column of the above table may be found in combination, but not those in different columns, except when the forms are alike. Thus,  $O$  cannot combine with  $\frac{mOm}{2}$ , but  $\infty O$  and  $\infty O \infty$  may be found in combination with any hemihedron. Further,  $mOm$ ,  $mO$ , and  $O$  are never found in combination with tetrahedral hemihedrons, nor  $mO \infty$  with pentagonal ones. When the +, the -, the right, or the left forms alone occur in combination there is no difficulty in distinguishing them from the corresponding holohedral forms, since by counting the planes possessing the same symbol it may be learned whether they are sufficiently numerous to constitute the holohedral form or only one-half this number.

By inspection of the crystal represented in Fig. 54 we detect two kinds of planes, A and B. The symbol of the A planes is  $\infty a : a : \infty a$ ; that of the B planes is  $ma : a : \infty a$ . There are six of the former present on the crystal and 12 of the latter. The holohedral form  $\infty O \infty$  possesses six planes; the holohedral form  $mO$  possesses 24. Hence the crystal is a combination of  $\infty O \infty$  and  $\pm \frac{mO}{2}$ .

When similar + and -, or right and left forms are in combination with one another, or when they are both present in combination with other forms, it becomes more difficult to distinguish between them and the holohedral form from which they are derived, for by a combination of the two hemihedrons all the planes of the original holohedron are represented, and it is only by a difference in size of the planes of the two hemihedrons or by some difference in their appearance that their true nature is recognized.

Figure 55 represents the combination of  $+\frac{O}{2}$  and  $-\frac{O}{2}$ . If

the planes of these two forms were of equal size, there would be no geometrical difference between the combination and the holohedral octahedron. The fact that four of the planes are small and four are large and that the small planes occupy the positions of the planes of one tetrahedron while the large ones occupy the positions of the planes of its congruent form, serves as the criterion by which this combination is distinguished from the corresponding holohedron.

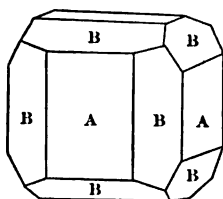


FIG. 54.—Combination of  $\infty O \infty$  and  $+ \left[ \frac{mO \infty}{2} \right]$

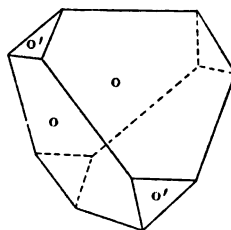


FIG. 55.—Combination of  $+ \frac{O}{2}$  and  $- \frac{O}{2}$

**Tetartohedrism of the Isometric System** (*Tetrahedral Pentagonal Dodecahedral Class*).—Only four new tetartohedral forms are possible in the isometric system, and these are rare. They are derived from the hexoctahedron by the development of three alternate planes in each alternate octant. They are known as positive and negative, right and left tetrahedral pentagonal dodecahedrons. Their symbols are  $\pm r \frac{mOn}{4}$  and  $\pm l \frac{mOn}{4}$ .

## CHAPTER VII.

### THE HEXAGONAL SYSTEM.

**Systems with One Principal Plane of Symmetry.**—The hexagonal and the tetragonal systems of crystals are characterized by possessing holohedrons with one principal plane of symmetry and several secondary planes. The principal plane is perpendicular to the secondary planes (Figs. 56 and 57), all of which intersect in a common line. The differences in the geometrical forms belonging to the two systems arise from the presence of four secondary planes of symmetry in the tetragonal crystals and six in the hexagonal crystals.

In each system the line of the intersection of the secondary planes of symmetry is taken as one of the crystallographic axes,

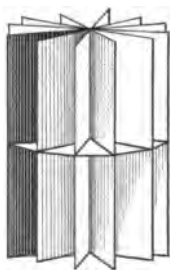


FIG. 56.

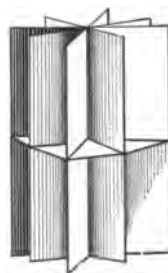


FIG. 57.

Distribution of planes of symmetry in systems with one principal plane.

and the intersections of the principal plane of symmetry with alternate secondary planes give the other axes. In each case the first axis differs from the others which are all equal. The two systems agree in possessing one axis which is not interchangeable with the others. In studying the crystals the latter is always held in a horizontal position. The analogies existing between the hexagonal and tetragonal system are so close that a careful study of one makes the study of the second very easy.

**Symmetry of the Hexagonal System.**—The hexagonal system includes all crystallographic forms possessing one principal plane of symmetry and six secondary planes and all the hemihedral and tetartohedral forms that may be derived from these. The secondary planes intersect each other in a common line and at an inclination of  $30^\circ$ . The principal plane of symmetry is perpendicular to these secondary planes (see Fig. 56).

**Crystallographic Axes.**—The lines chosen as the axes of the system are the intersection of the secondary planes with each other and the intersection of alternate secondary planes with the principal plane of symmetry. This selection yields three lines inclined to each other at angles of  $60^\circ$ , and all perpendicular to a fourth line. The latter is called the vertical axis and the other three the lateral axes (Fig. 58).

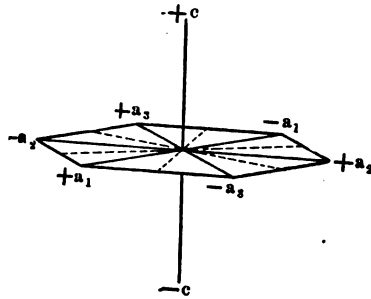


FIG. 58.—The system of axes in the hexagonal system.

The two ends of the vertical axis are separated by a plane of symmetry, hence these two ends must be equivalent. The lateral axes are also separated by planes of symmetry bisecting the angles between them, consequently the lateral axes must all be equivalent. But no plane of symmetry lies between the vertical and the lateral axes. These, therefore, are not equivalent; consequently, while the unities on the three lateral axes must be equal, the unity on the vertical axis has a different value.

**Designation of the Axes.**—As has already been explained, in studying crystals of the hexagonal system the lateral axes are held horizontally. The vertical axis thus becomes upright. The former are designated by the letter  $a$ , and the latter by the letter  $c$ . The scheme of the axes is  $a : a : a : c$ . This symbol indicates that the unities on the three lateral axes are the same and that the unity on the vertical axis is of some other value. The signs given to the axes are indicated in figure 58.

A model representing the ratios of the unity lengths of the



axes would be constructed of three straight wires of equal length intersecting each other at angles of  $60^\circ$ , and all perpendicular at their point of intersection to a fourth wire of different length.

**The Groundform and Axial Ratio.**—The groundform of the system is composed of planes cutting three of the axes at distances that are relatively the same as the unity distances.

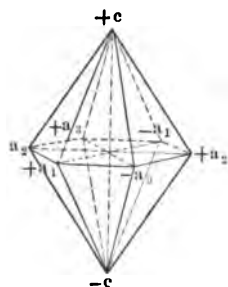


FIG. 59.—Groundform in hexagonal system.

Such planes cut two of the lateral axes at precisely the same distance from their point of intersection, and the third axis (which is the axis  $c$ ) at a longer or shorter distance from this point. (See Fig. 59.) The third lateral axis is cut at some distance other than unity—a distance which for the present we may represent as  $x$ . The symbol of a plane of this kind is  $a : a : xa : c$ .

When one of these planes is present, symmetry demands the presence of 11 others, which together form a *double hexagonal pyramid* (Fig. 59).

Since the unity length of  $c$  is different from the unity length of the  $a$  axes, it becomes necessary, before an hexagonal crystal can be studied, to determine the ratio between the two unities in order that a standard may be obtained to which to refer the intercepts of other planes on  $c$ . This unity is always recorded in terms of the unity on  $a$ . It, therefore, represents a ratio between the lengths at which a plane of the groundform cuts one of the lateral axes and the length at which it cuts the vertical axis.

Because it is a ratio between standard lengths on the axes, it is known as the axial ratio, and is written in the form of a ratio as  $a : c = 1 : 1.0999$ . This means that the unity on  $c$  is 1.0999 times the length of the unity on  $a$ . The value of the axial ratio depends primarily upon the groundform chosen, as the inclination of the planes of the groundform to the  $a$  and the  $c$  axes determines the ratio between the intercepts on these axes.

Let  $ABC$  and  $A'B'C'$  (Fig. 60) be the planes of two groundforms cutting the axes  $OA$ ,  $OB$ , and  $OC$  at  $A, B, C$ , and  $A', B', C'$ , respectively. The ratio between the lengths on  $OA$  and  $OB$  and on  $OC$  will be determined by the inclination of the planes to the

axes—the larger the angle made between the plane and the lateral axes, the larger will be the ratio between the intercepts on these axes and that on  $c$ ; or the larger will be the axial ratio or, in other words, the larger will be the unity on  $c$ .

### Determination of the

**Axial Ratio.**—Every substance that crystallizes in the hexagonal system *possesses a different groundform*. This groundform is always a double hexagonal pyramid, but the inclination of the faces differs for every different substance.

Hence, for every hexagonal mineral the axial ratio must be determined before the symbols of the planes occurring on its crystals can be calculated. Practically a form whose planes intercept the vertical axis and two of the lateral axes is assumed as the groundform. The relative distances at which one of its planes cuts the  $a$  and the  $c$  axes are calculated, after

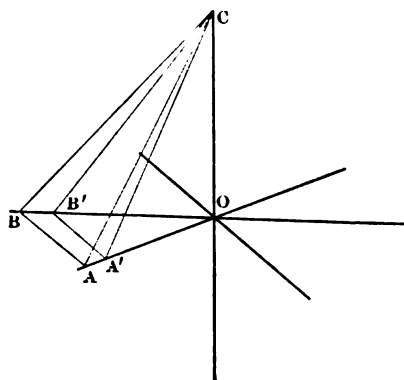


FIG. 60.

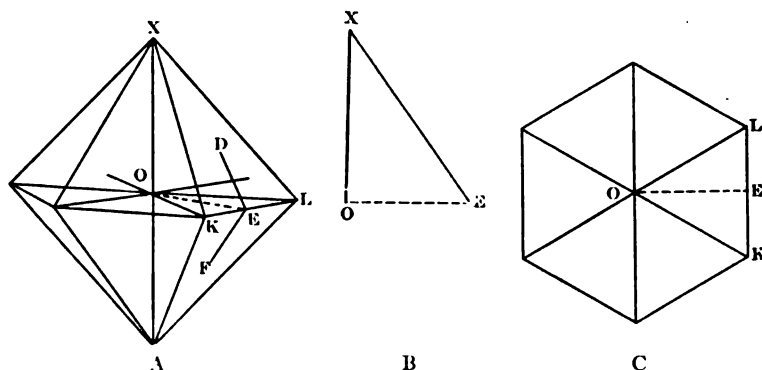


FIG. 61.

measurement of the proper interfacial angles, and this ratio becomes the axial ratio which is accepted by all crystallographers as representing the arbitrary value which shall be regarded as the ratio between the unities on these axes.

In figure 61 let A represent the groundform of a crystal the axial ratio of which is to be determined, and DEF, the interfacial angle between two faces measured at the termination of the line OE, drawn from the point of intersection of the axes perpendicular to the lateral edge. The distance OX = unity on  $c$ , and OK unity on  $a$ . Let B be a section through the lines OE and OX, and C, a section through the lateral axes. Represent one-half the measured angle by  $\beta$ .

In the triangle OEX,  $OX = OE \tan \beta$  (1)

In the triangle OEK,  $OE = OK \sin OKE$

Since  $OK = 1$  and  $OKE = 60^\circ$ , and  $\sin. 60^\circ = .866$

$$OE = .866$$

Substituting in (1) we have  $OX$  or  $c = .866 \tan \beta$

That is, the natural tangent of one-half the measured lateral interfacial angle on the ground-form pyramid multiplied by .866

is the axial ratio. In the mineral quartz ( $\text{SiO}_2$ ) the angle between the planes  $p$  and  $r$  is  $141^\circ 47'$  (Fig. 62). From this value the angle between  $p$  and  $z$  (below) is easily calculated as  $103^\circ 34'$ . One-half of this, or  $\beta = 51^\circ 47'$ , and its natural tangent is 1.27. Substituting in the equation above we have  $1.27 \times .866 = 1.0998$ , which is the axial ratio for all crystals of quartz.

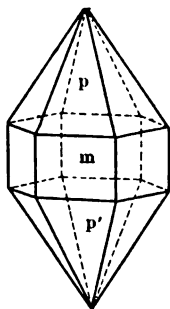


FIG. 62.—Crystal of quartz.

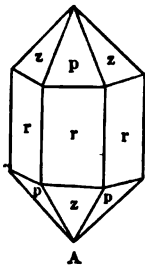
There are thus as many different axial ratios as there are substances crystallizing in the system. Consequently the axial ratio is a most important distinguishing characteristic of hexagonal substances. For nine important hexagonal minerals it is as follows:

Quartz	1 : 1.0999	Dolomite	1 : .8322	Nepheline	1 : .8389
Calcite	1 : .8543	Cinnabar	1 : 1.1453	Beryl	1 : .4988
Apatite	1 : .7346	Hematite	1 : 1.3650	Tourmaline	1 : .4481

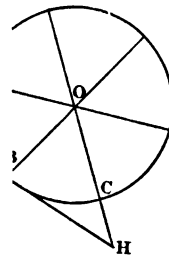
**The Intercept on the Third Lateral Axis.**—The ground-form in the hexagonal system is a double pyramid each of whose planes cuts two of the lateral axes at the same distance, assumed as unity, and the vertical axis at some different distance which

is also taken as unity. The symbol of its plane as given above (page 56) is  $a : a : xa : c$ . As usually written the symbol is  $a : a : c$ , or P.

The intercept on the third axis is generally omitted from the symbol because its value is determined by the intercepts on the other lateral axes, in such a way that  $x$  is always  $\frac{n}{n-1}$ , where  $n$  = the intercept on the first  $a$  axis. In the symbol  $a : a : xa : c$ ,  $n = 1$  and  $x = \frac{1}{0} = \infty$ . The plane is parallel to the third  $a$  and its complete symbol is  $a : a : \infty a : c$ . The  $\infty$  in the symbol is omitted because if the intercepts on two of the  $a$  axes are known the intercept on the third axis is also known.



Substitute this figure  
for FIG. 62, page 58.



is system the  
l the interven-

=0. Consequently, if two of the indices are known, the third can easily be deduced.

**Proof that the Intercept on the Third Lateral Axis is  $\frac{n}{n-1}$ , when the Intercepts on the Other Two Lateral Axes are  $n$  and Unity.**—In figure 63 let AO, BO, and OC represent the lateral axes intersecting at O in the center of a crystal, and let GH be the intersection of a pyramidal face with the plane of these axes. This plane intercepts the three axes at OG, OB, and OH, respectively, and OG, OB, and OH are its parameters on these axes. If  $OG = n$ , and  $OB = \text{unity} = 1$ , then OH, the parameter on the third axis, is  $\frac{n}{n-1}$ .

Inscribe a circle with O as the center and a radius equal to OB. Then will OA, OB, OC, etc., be the unity distances on these axes.

Draw the line BA from the unity distance on BO to the unity distance on AO. In the triangle AOB, the sides OA and OB are equal,  $\therefore$  the angles OAB and OBA are equal and each is  $60^\circ$ . The angle GAB is therefore  $120^\circ$ , as is also the angle AOH;  $\therefore$  AB is parallel to OH. Moreover, the triangle is equilateral and  $AB = OB = 1$ . The triangles GAB and GOH are similar  $\therefore$   $GA : GO :: AB : OH$ , or  $GA : n :: 1 : OH$ . But  $GA = GO - AO = n - 1$ ,  $\therefore$  the equation becomes  $n - 1 : n :: 1 : OH$ . Multiplying extremes and means gives  $OH \times n - 1 = n$ , or  $OH = \frac{n}{n - 1}$ . OH is the intercept on the axis OC, thus the parameter on the third axis is  $\frac{n}{n - 1}$ , when the parameters on the other two axes are  $n$  and 1.

**Symbols of Forms.**—In this system and in all of the succeeding systems the groundform is represented by P. Unity intercepts are not represented. Other intercepts on the  $c$  axis are written in front of P, and one of the lateral intercepts, provided these are not unity, is written after the P. In practice the smaller of the two lateral intercepts, when they are not equal, is indicated, the larger one being understood. Thus  $\frac{5}{4}a : a : 5a : 3c$  is written  $3 P \frac{5}{4}$ , it being understood that when one lateral intercept is  $\frac{5}{4}$  the other is 5.

### HOLOHEDRAL DIVISION.

(*Dihexagonal Bipyramidal Class.*)

**Symmetry of the Holohedral Forms.**—The holohedral forms of the hexagonal system, like the holohedral forms of the isometric system, possess the symmetry which is described as characterizing the system. In addition to the planes of symmetry described (Fig. 56) the holohedrons of this system possess a sixfold axis of symmetry perpendicular to the principal plane

of symmetry and six axes of binary symmetry at right angles to the sixfold axis, and a center of symmetry (Fig. 64). Figure 65 exhibits the symmetry relations projected on the plane passing through the lateral axes, i.e., upon the principal plane of symmetry.

**The Most General Form.**—The most general form of the system is composed of planes which cut the  $c$  axis at some distance other than unity and the three lateral axes at different

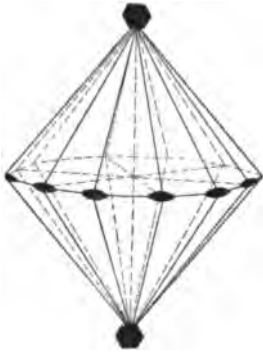


FIG. 64.

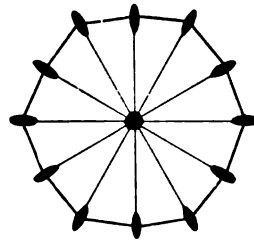


FIG. 65—Diagram illustrating the distribution of the elements of symmetry in hexagonal holohedrons.

distances. Since one of the latter may be considered as unity, the Naumann symbol of a plane of the most general form is  $na : a : \frac{n}{n-1}a : mc$ , where  $n$  is greater or less than  $\frac{n}{n-1}$ , provided neither is unity. When either  $n$  or  $\frac{n}{n-1}$  is unity, or when  $n = \frac{n}{n-1}$ , the plane will cut two of the lateral axes at the same distance, and hence cannot belong to the most general form in the system.

*The Dihexagonal Bipyramid.*—When  $n \geq \frac{n}{n-1}$ , and neither is unity, the planes cut one of the lateral axes at unity, another at  $n$  and the third at  $\frac{n}{n-1}$ . The presence of one of these planes

necessitates the presence of 11 others on one side of the principal plane of symmetry, and twelve on its opposite side. In all there are 24 planes on the form, one in each of the 24 compartments into which the symmetry planes of the system divide space.

The form thus produced is a double pyramid (Fig. 66) bounded by 24 similar scalene triangles. It is known as the *dihexagonal bipyramid*. The symbol of one of its

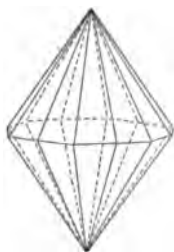


FIG. 66.—Dihexagonal bipyramid,  $mPn$  or  $hikl$ .

planes is  $na : a : \frac{n}{n-1} a = mc$ , or if the  $c$  axis is cut at the distance corresponding to the axial ratio, the symbol is  $na : a : \frac{n}{n-1} a : c$ , and the

symbol of the form is  $mPn$ , or  $Pn$ . Its indices are  $hikl$ . As has already been stated (page 59), it makes no difference whether the shorter or the longer of the two parameters that refer to the  $a$  axis is written after the  $P$  in the symbols of the forms in this system, as the value of each of these parameters depends upon the value of the other. The symbol  $2P\frac{3}{2}$  and  $2P3$  represent

exactly the same form, for  $n$  necessitates  $\frac{n}{n-1}$ , and *vice versa*.

If  $n=3$ , then  $\frac{n}{n-1} = \frac{3}{2}$ ; if  $n=\frac{3}{2}$ , then  $\frac{n}{n-1} = \frac{3/2}{1/2} = 3$ . It is simply custom that requires the use in the symbol of the smaller of the two parameters that are not unity.

**The Dihexagonal Series.**—A series of forms embraces those that may be derived from each other by a change in the value of a single parameter. The dihexagonal series is the series of forms that may be derived from the dihexagonal bipyramid by changing the parameter on the  $c$  axis.

The symbol of a plane of the most general form is  $na : a : \frac{n}{n-1} a : mc$ . By changing the parameter on  $c$  we may obtain the following symbols:

$$(1) \quad na : a : \frac{n}{n-1} a : mc.$$

$$(2) \quad na : a : \frac{n}{n-1} a : c.$$

$$(3) \quad na : a : \frac{n}{n-1} a : \frac{1}{m} c.$$

$$(4) \quad na : a : \frac{n}{n-1} a : \infty c.$$

$$(5) \quad na : a : \frac{n}{n-1} a = oc.$$

The planes represented by these symbols all bear the same relation to the  $a$  axes. They are, however, differently inclined to the  $c$  axis, and so produce forms that differ from the dihexagonal bipyramid according to the distance at which their planes cut the vertical axis.

The planes represented by the symbols (1), (2) and (3) are dihexagonal bipyramids. When either (2) or (3) occurs alone it cannot be distinguished from the most general form (1) unless the value of the unity on the  $c$  axis is known. When, however, this unity is known, a calculation of the values of the intercepts on  $c$  will discriminate between the three forms.

For example, one may have three dihexagonal bipyramids of the same substance. Measurement of their interfacial angles may show that a plane on one cuts the  $c$  axis at 2.1998 times the shortest distance at which it cuts the  $a$  axes, which distance is taken as the unity on these axes; that a plane on the second crystal intercepts the  $c$  axis at 1.0999 times the unity on  $a$ ; and that a plane on the third crystal intercepts  $c$  at .54995 times this unity on  $a$ . Evidently the three forms to which these planes belong are different and should be represented by different symbols. But there is nothing on the crystals themselves to guide us in the selection of the symbols. If, however, we know that the crystallized substance is quartz, the solution of the difficulty is easy. The axial ratio of quartz is 1 : 1.0999; i.e., the accepted relation between the unities on  $a$  and  $c$  is as 1 : 1.0999. The first of the three forms discussed is composed of planes that cut the  $c$  axis at twice this ratio, hence its symbol is  $2Pn$ . The symbol of the form on the second crystal is  $Pn$ , and that on the third crystal,  $1/2 Pn$ .



*The Dihexagonal Prism.*—The fourth symbol,  $na : a : \frac{n}{n-1}a : \infty c$  represents a plane that differs from a plane of  $mPn$  in that it is parallel to the  $c$  axis. The resulting form is a 12-sided prism (Fig. 67), the cross section of which is the same as the cross section of the dihexagonal bipyramid along the principal plane of symmetry. Its symbol is  $\infty Pn$  and its indices are *hiko*.

*Basal Pinacoid.*—The fifth symbol,  $na : a : \frac{n}{n-1}a : \infty c$ , stands for a plane that cuts the  $c$  axis at its point of intersection with the other axes, and the  $a$  axes all along their lengths. In other words, it is a plane parallel to the principal plane of symmetry. It is known as the basal pinacoid. Its symbol is  $\infty P$ , and indices,  $\infty 001$ .

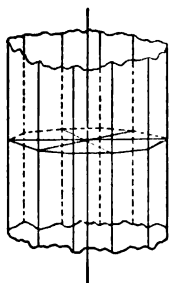


FIG. 67.—Dihexagonal prism,  $\infty Pn$  or *hiko*.

The symbol of this pinacoid requires that the plane shall cut one lateral axis at unity and the other two at  $n$  and  $\frac{n}{n-1}$ , respectively, and at the same time that it shall pass through the intersection of the axes. A plane that cuts either of the lateral axes at any distance, provided it passes through their intersection with the  $c$  axis, must cut them at all distances, hence it is not necessary to specify in the symbol any definite distance at which it cuts these  $a$  axes, since all distances are understood.

The basal pinacoid is a *pair of planes* perpendicular to the  $c$  axis. Two of them comprise the form, since the existence of one on either side of the plane of symmetry necessitates the existence of one more on its opposite side.

In nature the planes do not actually pass through the center of the crystals, since this is a physical impossibility. They are found at the terminations of the  $c$  axis, being parallel to the imaginary plane passing through the center of the crystal. (See Fig. 73.)

**Three Series of Holohedrons in the Hexagonal System.**—The three forms already noted, viz., the dihexagonal bipyramid, and prism, and the basal plane, constitute the dihexagonal

series. Besides this series there are two others possible in the system.

The dihexagonal series results when all of the lateral axes are cut at different distances. When two of the lateral axes are cut at the same distance, two other series arise, according to whether in the general symbol  $na : a : \frac{n}{n-1}a : mc$ ,  $n = \frac{n}{n-1}$ , or either  $n$ , or

$$\frac{n}{n-1} = \text{unity.}$$

When  $n = 1$ ,  $\frac{n}{n-1} = \infty$ ; and, conversely, when  $\frac{n}{n-1} = 1$ ,  $n = \infty$ .

When  $n = \frac{n}{n-1}$ ,  $n = 2$ : for clearing the equation of fractions we have  $n(n-1) = n$ . Dividing by  $n$  this is reduced to  $n-1 = 1$ , when  $n = 2$ .

These two changes in the general symbol are the only two that can possibly give rise to new forms from which new series may be derived by changes in the parameters on the  $c$  axis.

**Pyramids and Prisms of the First Order.**—A plane with the symbol  $a : a : \infty a : mc$ , cuts two of the lateral axes at unity and the  $c$  axis at some other distance than the unity on this axis. The third lateral axis is cut at  $\infty$ . When one plane with this symbol is present, symmetry requires the presence of eleven other similar planes (Fig. 68), resulting in a twelve-sided bipyramid composed of six isosceles triangular faces above the principal plane of symmetry, and the same number of similar faces below it. It is known as the hexagonal bipyramid. Its symbol is  $mP$ . The indices are  $h o h l$ .

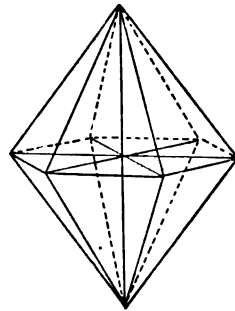


FIG. 68.—Hexagonal bipyramid of the first order,  $mP$  or  $hohl$ .

From an inspection of the cross section of this pyramid it will be noted that *the lateral axes terminate in the solid angles between four contiguous pyramidal faces*. Forms that bear this relation to the axes are known as *forms of the first order*.

From this pyramid a series of pyramids and a prism are derived by changing the intercept on  $c$ , as follows:

$$(1) a : a : \infty a : mc = mP$$

$$(2) a : a : \infty a : c = P$$

$$(3) a : a : \infty a : \frac{1}{m}c = \frac{1}{m}P$$

$$(4) a : a : \infty a : \infty c = \infty P$$

$$(5) a : a : \infty a : oc = oP$$

The symbols 1, 2, and 3 represent pyramids of the first order that differ from each other in the relative lengths of their vertical

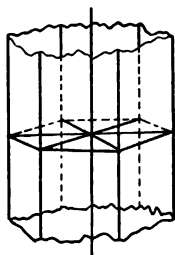


FIG. 69.—Hexagonal prism of the first order,  $\infty P$  or  $1010$ .

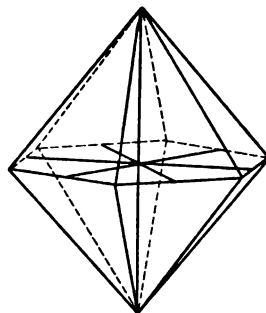


FIG. 70.—Hexagonal bipyramid of the second order,  $mP_2$ , or  $hh2hl$ .

dimensions. Symbol (2) represents the groundform of the system. The pyramid  $mP$  is more acute than this, and the pyramid  $\frac{1}{m}P$  more blunt.

Symbol (4) represents a prism of the first order (Fig. 69). This is composed of six faces with the axes terminating in their interfacial edges. Its symbol is  $\infty P$  or  $1010$ .

The plane  $a : a : \infty a : oc$  does not differ in character from the plane  $na : a : \frac{n}{n-1}a : oc$ , both, therefore, are represented by the same symbol; i.e.,  $oP$ .

**Pyramids and Prisms of the Second Order.**—The symbol  $2a : a : 2a : mc$ , or  $h h 2h l$ , designates a plane that cuts two of the lateral axes at twice unity, a third at unity, and the  $c$  axis at  $m$ .

This plane belongs to a bipyramid of 12 faces that is identical in appearance (see Fig. 70) with the hexagonal bipyramid of the first order. The difference between the two pyramids is simply in their relation to the axis. *Whereas in the form of the first order the axes terminate in the solid angles made by the conjunction of four faces, in the pyramid of the second order they terminate in the centers of the lateral edges.*

From the most general symbol of the series four other symbols are derived by changing the parameter on  $c$ :

$$(1) \ 2a : a : 2a : mc = mP_2$$

$$(2) \ 2a : a : 2a : c = P_2$$

$$(3) \ 2a : a : 2a : \frac{1}{m} = \frac{1}{m}P_2$$

$$(4) \ 2a : a : 2a : \infty c = \infty P_2$$

$$(5) \ 2a : a : 2a : oc = oP$$

Symbols (1), (2), (3), represent three bipyramids of the second order, in which the intercepts on the vertical axis are

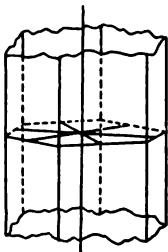


FIG. 71.—Hexagonal prism of the second order,  $\infty P_2$  or  $1120$ .

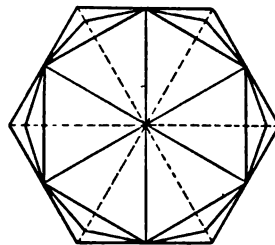


FIG. 72.—Diagram illustrating the relations of holohedral bipyramids and prisms to the axes in the hexagonal system.

different. Symbol (4) represents an hexagonal prism of the second order in which the axes terminate in the centers of the faces (see Fig. 71), and symbol (5) represents the basal pinacoid.

The relations of the pyramids of the three series to the lateral axes are seen in the diagram (Fig. 72), which represents cross sections through the pyramids of the three orders in the plane of the lateral axes. The inner hexagon corresponds to forms of the first order, the outer one to those of the second order, and the

dodecagon between the two hexagons to the dihexagonal forms. The three solid lines intersecting at the center of the figure are the axes.

In practice the distinction between forms of the first and second orders can be made only after the position of the axes has been determined. The selection of the axes is, moreover, merely a matter of convenience in many instances, those lines being chosen as the axes which will yield the simplest symbols for the forms occurring on the crystal under investigation. When the axes are once chosen, however, they remain fixed for all crystals of the substance and all planes must be referred to them.

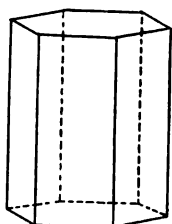


FIG. 73.—Hexagonal prism ( $\infty P$ ) terminated by basal plane ( $oP$ ).

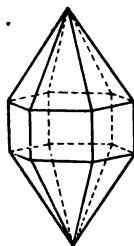


FIG. 74.—Combination of hexagonal prism and bipyramid of same order.

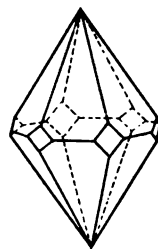


FIG. 75.—Combination of hexagonal prism and bipyramid of different orders.

**Closed and Open Forms.**—While each one of the several forms belonging to the isometric system will alone enclose space, this is not true of all the forms of the hexagonal system. The bipyramids may occur alone on a crystal, since each one completely encloses space. The prisms and the basal plane differ from the bipyramids in that neither can completely enclose space and therefore neither can exist alone. They occur either in combination with each other (Fig. 73) or in combination with some other form.

Forms that enclose space completely are often spoken of as *closed forms*, while those which do not completely enclose space are known as *open forms*. Crystals bounded by open forms cannot be represented by less than two symbols.

**Combinations.**—Combinations of holohedral forms in the hexagonal system are not as common as those of hemihedrons.

Where they occur they are easily understood, provided it is remembered that the axes determined for one form are the axes to which all the forms in the combination must be referred.

Figure 74 represents a combination of a prism and a pyramid belonging to the same order; figure 75, a prism and a pyramid

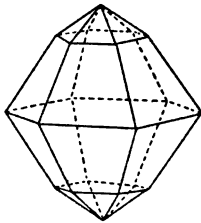


FIG. 76.—Combination of 2 hexagonal pyramids of the same order.

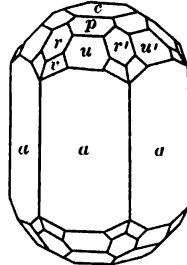


FIG. 77.—Crystal of beryl containing  $\infty P$  (*a*),  $P$  (*p*),  $2P_2$  (*r*),  $2P$  (*u*),  $3P^{\frac{3}{2}}$  (*v*), and  $oP$  (*c*).

of different orders; and figure 76, two pyramids of the same order. Figure 77 is more complicated. It represents a beryl ( $\text{Be}_3\text{Al}_2(\text{SiO}_3)_6$ ) crystal on which are the forms  $\infty P$  (*a*),  $P$  (*p*),  $2P_2$  (*r*),  $2P$  (*u*),  $3P^{\frac{3}{2}}$  (*v*), and  $oP$  (*c*).

### HEMIHEDRAL DIVISION.

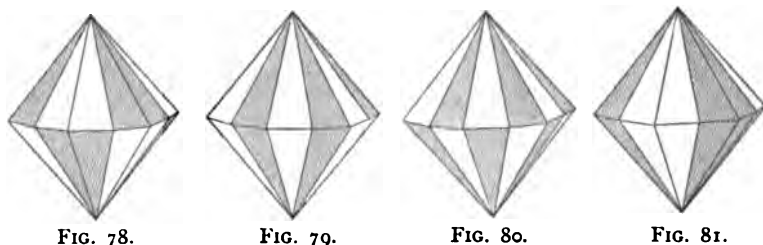
**Hemihedrism in the Hexagonal System.**—Although the hemihedral divisions of this system are as important as the holohedral division, we must limit our discussion to a few of the simplest of the hemihedral forms. These are extremely important, since some of the commonest minerals, like calcite ( $\text{CaCO}_3$ ), dolomite ( $\text{MgCaCO}_3$ ), and apatite ( $\text{Ca}_5\text{Cl}(\text{PO}_4)_3$ ), possess them.

**Possible Kinds of Hemihedrism.**—It will be remembered that the lateral axes are not equivalent to the vertical axis in this system. Hence the law of hemihedrism does not require that in the hemihedrons the ends of the vertical axis shall be terminated in the same way as the ends of the lateral axes. It demands simply that the new hemihedral forms shall affect the two ends of the vertical axis similarly and the several ends of the lateral axes.

The number of possible ways by which one-half the planes of the general form in this system may be combined is even greater than in the isometric system. In only four cases, however, does the result comply with the conditions of hemihedrism. These are indicated in the accompanying figures, in which all the white planes are represented as surviving or all the shaded planes.

The four classes of hemihedrism derived by these four methods possess different grades of symmetry. The names applied to them are:

*The Rhombohedral class* (Fig. 78) in which the hemihedrons possess all the faces of the holohedrons that lie in alternate dodecants.



Figures illustrating the four possible methods of derivation of the hemihedrons in the hexagonal system. Fig. 78, rhombohedral; 79, pyramidal; 80, trapezohedral; 81, trigonal.

*The Pyramidal class* (Fig. 79) in which the alternate pairs of planes meeting in the lateral edges survive.

*The Trapezohedral class* (Fig. 80) in which the planes that survive occupy the alternate compartments of the 24 into which space is divided by the planes of symmetry.

*The Trigonal class* (Fig. 81) in which all the planes in alternate sextants survive.

Only rhombohedral and pyramidal hemihedrons have been found on crystals in complete forms that are geometrically distinct from holohedrons, but hemimorphic forms of the trigonal class are also known.

**Rhombohedral Hemihedrons** (*Trigonal Scalenohedral Class*).—The rhombohedral hemihedrons may be regarded as derived from holohedrons by the suppression of all the planes

that lie in alternate dodecants. The new forms thus derived lose the principal plane of symmetry, the three secondary planes passing through the lateral axes, and the three axes of binary symmetry lying between these crystallographic axes. The remaining elements of symmetry, viz., the three secondary planes lying between the lateral axes, the three axes of binary symmetry coinciding with these crystallographic axes, and the center of symmetry survive. The axis of sixfold symmetry becomes an axis of threefold symmetry. This coincides with the vertical crystal axis. (Fig. 82.) In Fig. 82, which is a diagrammatic representation of these symmetry relations, the heavy broken line indicates the disappearance of the principal plane of symmetry. The lighter broken lines indicate the positions of the secondary planes that have disappeared, and the light unbroken lines the position of those retained.

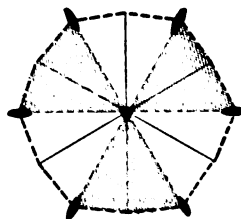


FIG. 82.—Diagram illustrating the distribution of the elements of symmetry in rhombohedral hemihedrons.

Only two new geometrical hemihedrons are possible in this class.

*The Scalenohedrons.*—The extension of the planes of the dihexagonal pyramid occupying alternate dodecants yields a form bounded by twelve similar scalene triangles. (Figs. 83 and 84.) These intersect in two kinds of interfacial edges extending from the ends of the vertical axis to the plane of the lateral axes, and a third kind connecting the ends of the lateral axes in a zig-zag line. The lateral axes terminate in the centers of these edges. The forms are known as *hexagonal scalenohedrons*. Their symbols are  $\pm \frac{mPn}{2}$ . Two congruent scalenohedrons may be derived from every dihexagonal bipyramid. They are distinguished as the positive (Fig. 83) and the negative (Fig. 84) forms. Their indices are  $\kappa(h\bar{i}kl)$  and  $\kappa(ihkl)$ .

*The Rhombohedrons.*—By suppressing every alternate plane composing the pyramid of the first order and extending the remaining planes two new congruent forms are produced, each bounded by six similar rhombs. These possess two kinds of solid



angles, of which two are polar and four lateral. The polar angles are larger or smaller than the other four depending upon the value of the intercepts on  $c$ . When the axial ratio is  $1 : \sqrt{1.5}$

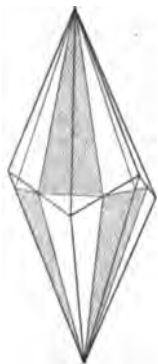


FIG. 83.—Positive hexagonal scalenohedron,  $+\frac{mPn}{2}$  or  $\kappa(hikl)$ .

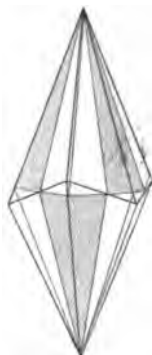


FIG. 84.—Negative hexagonal scalenohedron,  $-\frac{mPn}{2}$  or  $\kappa(h\bar{i}kl)$ .

or  $1 : 1.22474$ , the polar and lateral angles on the unity rhombohedron  $\left(\frac{P}{2}\right)$  are equal. The new forms are called the positive (Fig. 85) and the negative (Fig. 86) rhombohedron—

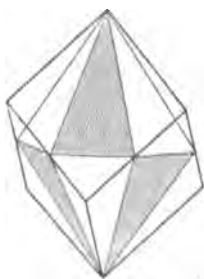


FIG. 85.—Positive rhombohedron,  $+\frac{mP}{2}$  or  $+R$ , or  $\kappa(hohl)$ .

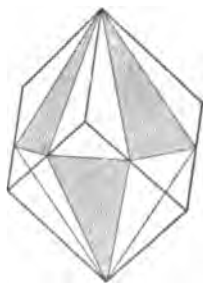


FIG. 86.—Negative rhombohedron,  $-\frac{mP}{2}$  or  $-R$ , or  $\kappa(chhl)$ .

the latter being the one that turns an upper edge toward the observer. Their symbols are  $+\frac{mP}{2}$  and  $-\frac{mP}{2}$ , and their in-

dices  $\kappa(hohl)$  and  $\kappa(ohhl)$ . In them the axes terminate in the centers of the six lateral edges.

**Short Symbols for the Scalenohedron and the Rhombohedron.**—The two rhombohedral hemihedral forms are so frequently met with in nature that many crystallographers use simpler symbols than those given above to represent them.

The rhombohedron derived from  $P$  is represented by  $R$ . That derived from  $mP$  is  $mR$ , etc. The scalenohedron is represented by  $Rp$ ,  $mRp$ , etc., in which  $R$  and  $mR$  signify the rhombohedron with the same lateral edges as the scalenohedron (rhombohedron of the middle edges, Fig. 87) and  $p$  the distance at which the scalenohedral faces intersect the vertical axis in terms of the corresponding intersection of the planes of the rhombohedron of the middle edges. The  $m$  and  $p$  in this symbol, therefore, do not correspond to

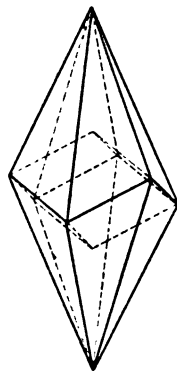


FIG. 87.—Hexagonal scalenohedron, enclosing rhombohedron of the middle edges.

$m$  and  $n$  in the symbol  $\frac{mPn}{2}$ . The symbols may be transformed from one system into the other with the aid of the following equations:\*

$$(1) \frac{mPn}{2} = \frac{m(2-n)}{n} R \frac{n}{2-n}.$$

$$\text{Thus } \frac{3}{2} P \frac{3/2}{2} = R^3.$$

$$(2) mRp = \frac{m p P}{2} \frac{2p}{p+1}.$$

$$\text{Thus } \frac{2}{2} R^2 = \frac{4}{2} P \frac{4/3}{2}.$$

**Combinations.**—The combinations of the pyramids and prisms of the first and second orders with the rhombohedrons are at first confusing, but a little consideration of the position of the axes will nearly always serve to make them clear.

Figure 88 represents a combination of  $-R$  with the prism of

\* For proof of the correctness of these equations see Groth: *Physikalische Krystallographie*, 1885, p. 342 and p. 348.

the first order; figure 89, a combination of  $+R$  with the prism of the second order; and figure 90, that of a  $+R$  with a  $-R$ . If the

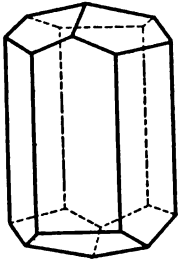


FIG. 88.—Combination of negative rhombohedron ( $-R$ ) with prism of the first order ( $\infty P$ ).

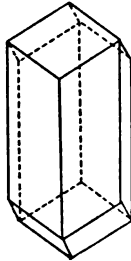


FIG. 89.—Combination of positive rhombohedron ( $+R$ ) with prism of the second order ( $\infty P_2$ ).

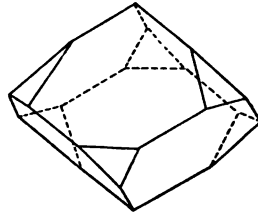


FIG. 90.—Combination of positive and negative rhombohedrons.

positions of the axes are fixed with reference to the rhombohedrons in these crystals, the nature of the prisms is easily recognized. The crystals represented in figures 91 and 92 are

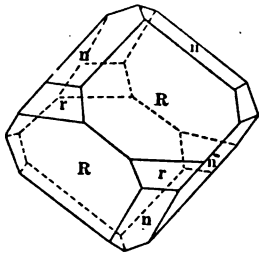


FIG. 91.—Hematite ( $\text{Fe}_2\text{O}_3$ ) crystal containing  $R(R)$ ,  $-R(r)$  and  $-\frac{1}{2}R(n)$ .

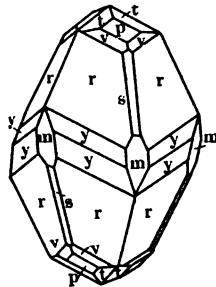


FIG. 92.—Crystal of calcite containing  $R(p)$ ,  $\frac{5}{2}R(s)$ ,  $4R(m)$ ,  $R^3(r)$ ,  $R^5(y)$ ,  $R^{\frac{1}{2}}(v)$ , and  $\frac{1}{4}R^3(t)$ .

more complicated. The first contains planes of the forms  $R(R)$ ,  $-R(r)$ , and  $-\frac{1}{2}R(n)$ . Figure 92 represents a crystal of calcite ( $\text{CaCO}_3$ ), with the forms  $R(p)$ ,  $\frac{5}{2}R(s)$ ,  $4R(m)$  and the scalenohedrons  $R^3(r)$ ,  $R^5(y)$ ,  $R^{\frac{1}{2}}(v)$ , and  $\frac{1}{4}R^3(t)$ .

**Pyramidal Hemihedrons** (*Hexagonal Bipyramidal Class*).—

The pyramidal hemihedrons may be derived from the holohedrons by extending the alternate pairs of planes meeting in the lateral edges. They are characterized by the presence of a principal plane of symmetry, an axis of sixfold symmetry, and a center of symmetry. The secondary planes of symmetry and the six axes of binary symmetry of the holohedrons are lost (Fig. 93).

The only holohedrons from which new pyramidal hemihedrons may be derived are the dihexagonal bipyramids and prisms.

*The Pyramids of the Third Order.*—The dihexagonal bipyramid gives rise to two bipyramids of the third order which



FIG. 93.—Diagram illustrating the distribution of the elements of symmetry in pyramidal hemihedrons.

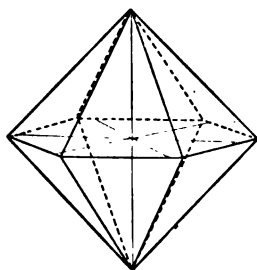


FIG. 94.—Positive hexagonal bipyramid of the third order,  $+\left[\frac{mPn}{2}\right]$  or  $\pi(h\bar{k}l)$ .

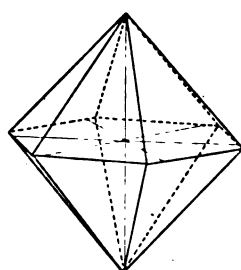


FIG. 95.—Negative hexagonal bipyramid of the third order,  $-\left[\frac{mPn}{2}\right]$  or  $\pi(hkil)$ .

are like the pyramids of the first and second orders in possessing 12 similar triangular faces and a hexagonal cross section (Figs. 68 and 70). They differ from these bipyramids, however, in their relation to the lateral axes. In the bipyramid of the third order the axes terminate in the lateral edges somewhere between the centers of the edges and the solid angles. The two forms are congruent and their symbols are  $+\left[\frac{Pn}{2}\right]$ ,  $+\left[\frac{mPn}{2}\right]$  (Fig. 94),  $-\left[\frac{Pn}{2}\right]$ ,  $-\left[\frac{mPn}{2}\right]$  (Fig. 95), etc., and their indices are  $\pi(h\bar{k}l)$  and  $\pi(hkil)$ .

*The Prisms of the Third Order.*—In the same way the dihexagonal prisms yield prisms of the third order (Fig. 96), which,

when observed alone, are not distinguishable by sight from the prisms of the first order. When in combination with other prisms, however, they are easily recognizable from the fact that the lateral axes terminate neither in the interfacial edges between contiguous planes nor in the centers of these planes. Their symbols are  $\pm \left[ \begin{smallmatrix} \infty Pn \\ 2 \end{smallmatrix} \right]$  and their indices  $\pi(hiko)$  and  $\pi(hkio)$ .

**The Relations of the Pyramids and Prisms of the Three Orders.**—The pyramids of the three orders are impossible to distinguish by the eye alone when either occurs singly. If the axial ratio is known, a measurement of the lateral interfacial angle and a calculation from this of the ratio between the distances at which the pyramidal planes cut the  $a$  and  $c$  axes will determine the question. If this ratio is the same as the axial ratio, or a multiple of it, the pyramid is of the second order ( $c = OE \tan \beta$ , where  $OE = 1$ . Compare p. 58.)

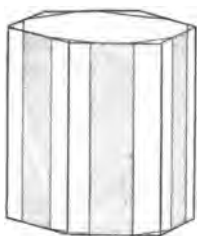


FIG. 96—Hexagonal prism of the third order,  $+ \left[ \begin{smallmatrix} \infty Pn \\ 2 \end{smallmatrix} \right]$ , or  $\pi(hiko)$ .

If the axial ratio is different from the determined ratio, the pyramid is of the first or third order. If it is .866 that of the determined ratio, or some multiple of .866, the pyramid is of the first order (compare p. 58). If the determined ratio is not the same as the axial ratio or some multiple of it, and is not .86, 1.7, or 2.5 of it, the pyramid is of the third order.

The three orders of prisms are impossible to distinguish either by the eye or by measurement when they occur alone, even if the axial ratio is known.

When in combination there is little difficulty in discriminating between the different orders. If the order of any form in the combination is known, it is only necessary to locate the lateral axes to determine the orders of all the prisms and pyramids in the combination. The relation of the orders to the axes is indicated in the subjoined figure (Fig. 97) in which the heaviest line indicates the position of the prism of the third order.

**The Trapezohedral Hemihedrons** (*Hexagonal Trapezohe-dral Class*).—In the hemihedrons of this class all the planes of

symmetry and the center of symmetry have disappeared. The forms therefore possess no pairs of parallel planes. All the axes of symmetry remain (Fig. 98).

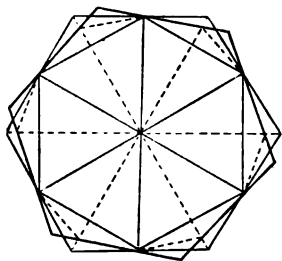


FIG. 97.—Diagram illustrating the relations of the prisms and pyramids of the three orders to the hexagonal axes.



FIG. 98.—Diagram illustrating the distribution of the elements of symmetry in hexagonal trapezohedral hemihedrons.

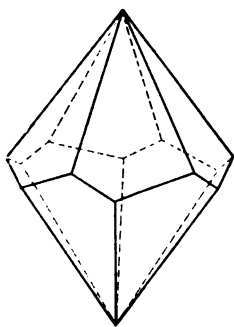


FIG. 99.—Right hexagonal trapezohedron,  $r \frac{mPn}{2}$  or  $\tau(hikl)$ .

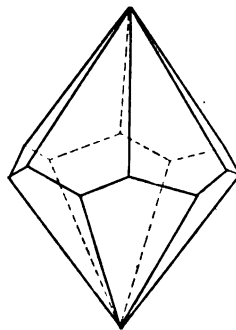


FIG. 100.—Left hexagonal trapezohedron,  $l \frac{mPn}{2}$ , or  $\tau(kihl)$ .

Two new forms are possible. These are known as *hexagonal trapezohedrons*. They are enantiomorphous (see page 83), and are represented by the Naumann symbols  $r \frac{mPn}{2}$ , and  $l \frac{mPn}{2}$ . The  $r$  and  $l$  signify right (Fig. 99) and left (Fig. 100).

Since no crystals bearing forms of this class have been observed, it is needless to discuss them in this book.

### The Trigonal Hemihedrons (*Ditrigonal Bipyramidal Class*).

—These forms have never been observed except on hemimorphic crystals. The theoretical forms possess one principal and three secondary planes of symmetry, an axis of trigonal symmetry, and three binary axes of symmetry. Three of the secondary planes of symmetry and three of the axes of binary symmetry belonging to the holohedrons have disappeared, as has also the center of symmetry. The axis of sixfold symmetry has become an axis of threefold symmetry (Fig. 101). As observed in connection with hemimorphism, the principal plane of symmetry and the three axes of binary symmetry are lost and the axis of trigonal symmetry becomes polar.

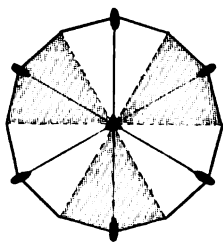


FIG. 101.—Diagram illustrating the distribution of the elements of symmetry in trigonal hemihedrons.

The forms of this class are characterized by triangular or triangle-like cross sections.

*The Trigonal Prisms and Bipyramids.*—These hemihedrons may be derived from the prisms and pyramids of the first order by the suppression of the planes in alternate sextants. The

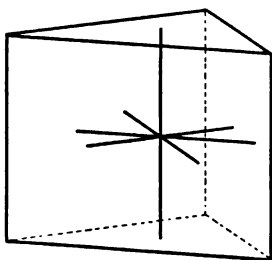


FIG. 102.—Positive trigonal prism,  $+\frac{\infty P}{2}$ , or *hoho*.

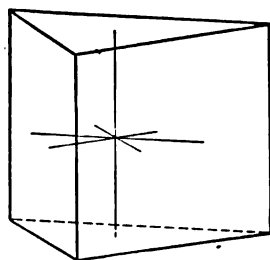


FIG. 103.—Negative trigonal prism,  $-\frac{\infty P}{2}$ , or *ohho*.

results are two congruent prisms and two congruent bipyramids with cross sections that are equilateral triangles. Two lateral axes terminate in each of the planes of the prism and in each lateral edge of the pyramid in such a manner that the distance between the polar edges is divided into three equal spaces.

The new forms are known as the trigonal prisms and bipyramids. There are two of each with the symbols  $+\frac{\infty P}{2}$  (Fig. 102) and  $-\frac{\infty P}{2}$  (Fig. 103), and  $+\frac{mP}{2}$  (Fig. 104) and  $-\frac{mP}{2}$  (Fig. 105). Their corresponding indices are *hoho*, *ohho* and *hohl* and *ohhl*.

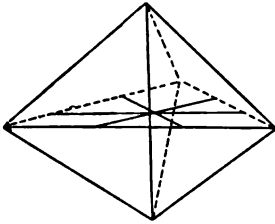


FIG. 104.—Positive trigonal bipyramid  $+\frac{mP}{2}$ , or *hohl*.

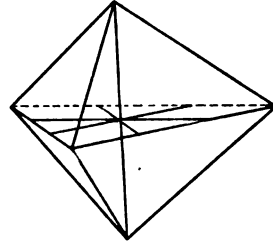


FIG. 105.—Negative trigonal bipyramid,  $-\frac{mP}{2}$ , or *ohhl*.

*The Ditrigonal Prisms and Bipyramids.*—Similarly the dihexagonal prisms and bipyramids yield four new forms, two congruent ditrigonal prisms and two congruent ditrigonal bipyramids. These possess two kinds of polar edges and their cross sections

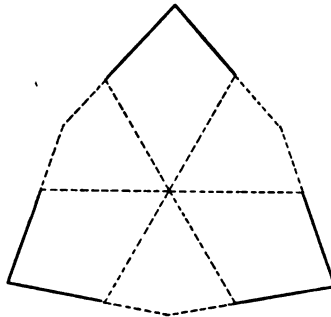


FIG. 106.—Diagram illustrating relation of planes in ditrigonal pyramid and prisms to the axes in the hexagonal system.

are six-sided with the sides arranged in pairs so as to produce a triangle-like outline. The crystallographic axes in the prisms terminate at points between the centers of the faces and the blunt vertical edges. In the bipyramids they terminate at the corresponding points on the lateral edges (Fig. 106).



The symbols of the prism are  $\pm \frac{\infty Pn}{2}$  (Figs. 107 and 108) and  $hiko, ihko$ , and of the bipyramids  $\pm \frac{mPn}{2}$  (Figs. 109 and 110) and  $hikl, ihkl$ .

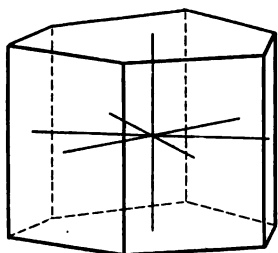


FIG. 107.—Positive ditrigonal prism,  $+\frac{\infty Pn}{2}$ , or  $hik^0$ .

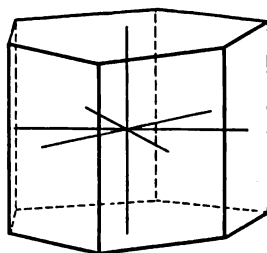


FIG. 108.—Negative ditrigonal prism,  $-\frac{\infty Pn}{2}$ , or  $ihk^0$ .

**Combinations with Hemimorphism (Ditrigonal Pyramidal Class).**—As has already been stated, the forms of this class are known only in combination with hemimorphism. Consequently only the upper or lower half of each form is present on any one crystal. In the case of the prisms, however, there is no geometrical

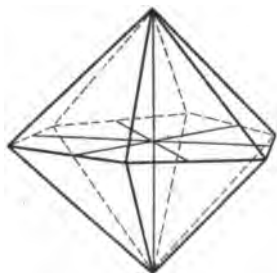


FIG. 109.—Positive ditrigonal bipyramid,  $+\frac{mPn}{2}$ , or  $hikl$ .

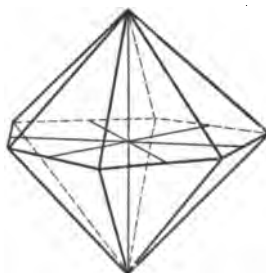


FIG. 110.—Negative ditrigonal bipyramid,  $-\frac{mPn}{2}$ , or  $ihkl$ .

difference noticeable between the complete hemihedral forms and those that are hemimorphic, since each plane belongs at the same time to both upper and lower dodecants. In the case of the bipyramids, on the other hand, half of the planes exist either at the upper pole or at the lower one. Consequently there are

four hemimorphic forms of the bipyramid possible. Because the planes on the hemimorphic pyramids are only  $1/4$  the total number of planes on the corresponding holohedron their

symbols become  $+\frac{mP}{4}u$ , (*hohl*),  $+\frac{mP}{4}l$  (*ohhl*),  $-\frac{mP}{4}u$  (*ohhl*) and  $-\frac{mP}{4}l$  (*hohl*), in which the letters *u* and *l* after the Naumann symbols indicate upper and lower.

The trigonal hemihedrons are easily recognized in combinations by their cross sections.

The best illustration of this type of hemihedrism is shown by the mineral tourmaline (a complex boro-silicate); figure 111 represents the combination of

$+\frac{P}{4}u$  (*P*),  $+\frac{P}{4}l$  (*P'*),  $-\frac{2P}{4}u(o')$ ,  $-\frac{oP}{2}l$  (*c*),  $\frac{1}{2}\frac{P}{4}l$  (*n*),  $\infty P_2(s)$ ,  $+\frac{3P_{3/2}}{4}u$  (*t*) and  $-\frac{\infty P}{2}(l)$ .

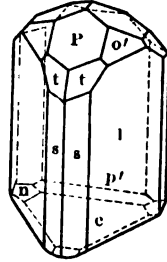


FIG. 111.—Crystal of tourmaline showing hemimorphic combinations of trigonal hemihedrons.

#### TABULAR SUMMARY OF HEXAGONAL HEMIHEDRONS.

(See explanation under list of isometric hemihedrons, page 51.)

Holo- hedrons	Hemihedrons				
	Rhombohedral	Pyramidal	Trapezohedral	Trigonal Hemimorphic	
<i>mPn</i>	$\pm \frac{mPn}{2}$ or <i>mRp</i>	$\pm \left[ \frac{mPn}{2} \right]$	<i>r.l.</i> $\frac{mPn}{2}$	$\pm \frac{mPn}{4}$	<i>u.l.</i>
$\infty Pn$	$\infty Pn$	$\pm \left[ \frac{\infty Pn}{2} \right]$	$\infty Pn$	$\pm \frac{\infty Pn}{2}$	
<i>mP<sub>2</sub></i>	<i>mP<sub>2</sub></i>	<i>mP<sub>2</sub></i>	<i>mP<sub>2</sub></i>	<i>mP<sub>2</sub></i>	
$\infty P_2$	$\infty P_2$	$\infty P_2$	$\infty P_2$	$\infty P_2$	
<i>mP</i>	$\pm \frac{mP}{2}$ , or <i>R</i>	<i>mP</i>	<i>mP</i>	$\pm \frac{mP}{4}$	<i>u.l.</i>
$\infty P$	$\infty P$	$\infty P$	$\infty P$	$\pm \frac{\infty P}{2}$	
<i>oP</i>	<i>oP</i>	<i>oP</i>	<i>oP</i>	<i>oP</i>	

**Combinations of Hemihedrons.**—The statements made with reference to the combinations of hemihedrons in the isometric system (pp. 51–52) apply equally well to those of the hexagonal and the succeeding systems. Only those hemihedrons may combine that have the same grade of symmetry; i.e., of the forms indicated above, only those may be found in combination that are represented by the symbols in the same vertical columns.

### TETARTOEDRAL DIVISION.

**Tetartohedrism of the Hexagonal System.**—The tetartohedral forms of this system, like the hemihedral forms, are of great theoretical importance because the most widely spread of all minerals, quartz ( $\text{SiO}_2$ ), often exhibits well-characterized tetartohedral planes. Moreover, they are of considerable practical interest because of the fact that there is a close relation existing between certain important physical properties of tetartohedral crystals and the planes occurring on them.

There are three classes of tetartohedrons in this system distinguished from one another by symmetry. They are known as *Trigonal*, *Trapezohedral*, and *Rhombohedral tetartohedrons* because their characteristic forms are trigonal bipyramids and prisms, trapezohedrons and rhombohedrons. They naturally possess a different grade of symmetry from the hemihedral forms of the same names, and therefore are designated as of different orders in the case of the bipyramids, prisms, and rhombohedrons. The tetartohedral trapezohedrons have three-sided polar angles, and therefore are distinguished from the corresponding hemihedral forms, which have six-sided polar angles, by prefixing the adjective trigonal to the name of the form. Only the trapezohedral tetartohedrons will be discussed.

**Trapezohedral Tetartohedrons** (*Trigonal Trapezohedral Class*).—Only ten forms of this class are different geometrically from holohedrons and hemihedrons. Of these four are trigonal trapezohedrons, two are ditrigonal prisms, two are trigonal prisms, and two are trigonal pyramids. The only elements of symmetry in them are an axis of threefold symmetry coinciding with the vertical crystallographic axis and three polar axes

of binary symmetry which coincide with the lateral crystallographic axes. The loss of the center of symmetry means the absence of pairs of parallel planes. (Fig. 112.)

The forms may be regarded as being derived from the holohedrons by applying to them at the same time the conditions of rhombohedral and trapezohedral hemihedrism (Fig. 113); or they may be thought of as being derived from the rhombohedral hemihedrons by applying to them the condition of trapezohedral hemihedrism, or *vice versa*.



FIG. 112.—Diagram illustrating distribution of symmetry elements in trapezohedral tetartohedrons.

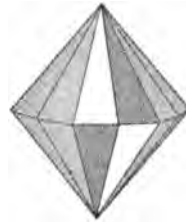


FIG. 113.—Application of rhombohedral and trapezohedral hemihedrism to the dihexagonal bipyramid.

**Trigonal Trapezohedrons.**—The trigonal trapezohedrons contain six trapezoid faces that correspond to six faces of the dihex-

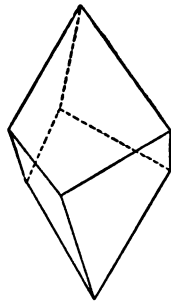


FIG. 114.—Positive right trigonal trapezohedron,  $+r \frac{mPn}{4}$ , or  $hikl$ .

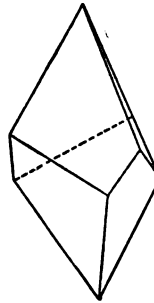


FIG. 115.—Positive left trigonal trapezohedron,  $+l \frac{mPn}{4}$ , or  $k\bar{i}h\bar{l}$ .

agonal bipyramid. They meet in six equal polar edges and in zig-zag lateral edges made up of long and short lines. Three

lateral axes with the same sign terminate in the centers of the long lateral edges, and the other three in the centers of the short edges. Since each form contains but one-fourth the planes of the holohedron, there are in all four forms derivable from each holohedron (see Fig. 113). These are designated as positive (+), negative (-), right (*r*), and left (*l*). The + and - forms are congruent and the *r* and *l* forms are *enantiomorphous*; i.e., they are symmetrical with respect to one another and neither can be so revolved that its faces shall be parallel to the faces of the other.

The Naumann symbols are  $+r\frac{mPn}{4}$  (Fig. 114),  $-r\frac{mPn}{4}$ ,  $+l\frac{mPn}{4}$  (Fig. 115), and  $-l\frac{mPn}{4}$ . The corresponding indices are *hikl*, *ihkl*, *kihl* and *khil*.

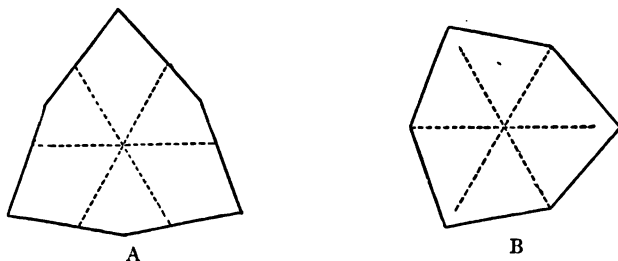


FIG. 116.—Diagram illustrating relation of planes to axes in hemihedral (A) and tetartohedral (B), ditrigonal prisms and pyramids.

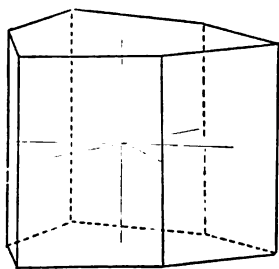


FIG. 117.—Right ditrigonal prism of the second order,  $r\frac{\infty Pn}{2}$ , or *hiko*.

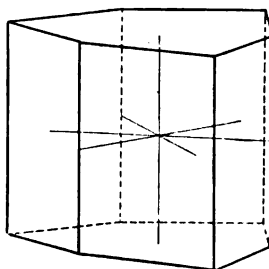


FIG. 118.—Left ditrigonal prism of the second order,  $l\frac{\infty Pn}{2}$ , *khio*.

*The Ditrigonal Prisms of the Second Order.*—The dihexagonal prisms yield two ditrigonal prisms with the symmetry of trapezohedral tetartohedrim. These are geometrically similar to the hemi-

hedral ditrigonal prisms. They differ from them, however, in the relation of their planes to the axes (see Fig. 116). Because of this difference the forms of this class are known as of the second order, and their symbols are written  $r \frac{\infty Pn}{2}$  (Fig. 117) and  $l \frac{\infty Pn}{2}$  (Fig. 118), or *hiko* and *kiho*.

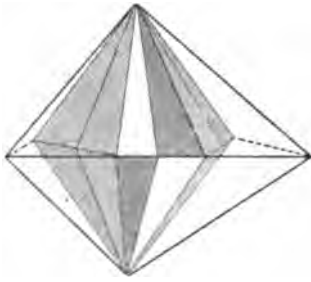


FIG. 119.—Right trigonal bipyramid of the second order,  $r \frac{mP_2}{2}$ , or *hhzhl*.

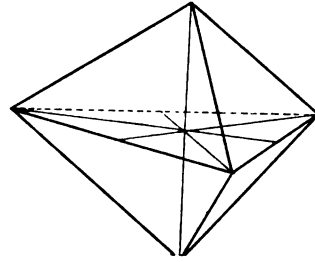


FIG. 120.—Left trigonal bipyramid of the second order,  $l \frac{mP_2}{2}$ , or *zhhl*.

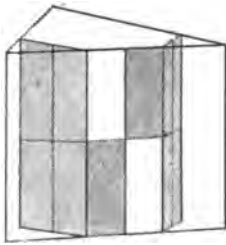


FIG. 121.—Right trigonal prism of the second order,  $r \frac{\infty P_2}{2}$ , or *hhzho*.

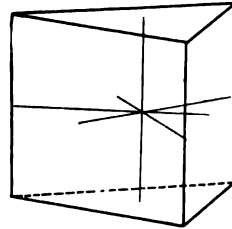


FIG. 122.—Left trigonal prism of the second order,  $l \frac{\infty P_2}{2}$ , or *zhho*.

*Trigonal Bipyramids and Prisms of the Second Order.*—The hexagonal bipyramids and prisms of the second order yield trigonal bipyramids and prisms of the second order which differ from the corresponding hemihedral trigonal forms in their positions with respect to the axes—the relation of their faces to the axes being the same as in the hexagonal pyramid of the second order; viz., in the prism each face is perpendicular to one

axis, and in the pyramids each lateral edge is at right angles to an axis (see Figs. 120 and 122).

When in combination with other forms the tetartohedral forms are easily recognized by their positions. To distinguish them from other similar geometrical forms they are represented by  $r \frac{mP_2}{2}$ ,  $l \frac{mP_2}{2}$ , and  $r \frac{\infty P_2}{2}$  and  $l \frac{\infty P_2}{2}$  (Figs. 119, 120, 121, and 122).

**Combinations.**—The mineral exhibiting these forms in greatest perfection is quartz ( $\text{SiO}_2$ ). Two crystals of this substance are shown in figures 123 and 124. Crystals showing right forms are referred to as right crystals, and those showing left forms are known as left crystals. The former possess the power of rotating the plane of polarized light to the right, and the latter the power of turning it to the left. Since this effect of quartz

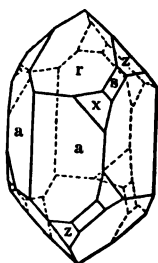


FIG. 123.

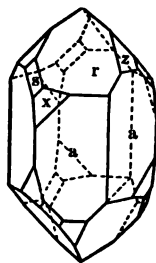


FIG. 124.

Crystals of right-handed (Fig. 123) and left-handed (Fig. 124) quartz crystals containing  $\infty P(a)$ ,  $+\frac{P}{2}(r)$ ,  $-\frac{P}{2}(z)$ ,  $+\frac{2P_2}{2}$  and  $+r \frac{6P_6/5}{4}$  ( $s$  and  $x$  in Fig. 123), and  $-\frac{2P_2}{2}$  and  $+l \frac{6P_6/5}{4}$  ( $s$  and  $x$  in Fig. 124).

upon polarized light is made use of in the manufacture of certain optical instruments, the ability to recognize right and left forms is extremely valuable. The forms exhibited by the two crystals figured are:  $\infty P(a)$ ,  $+\frac{P}{2}(r)$ ,  $-\frac{P}{2}(z)$ ,  $+\frac{2P_2}{2}$  ( $s$  in Fig. 123),  $-\frac{2P_2}{2}$  ( $s$  in Fig. 124),  $+r \frac{6P_6/5}{4}$  ( $x$  in Fig. 123),  $+l \frac{6P_6/5}{4}$  ( $x$  in Fig. 124).

## CHAPTER VIII.

### THE TETRAGONAL SYSTEM.

#### Similarity Between Tetragonal and Hexagonal Systems.

—Reference has already been made (p. 54) to the similarity between tetragonal and hexagonal forms in consequence of the existence in them of one principal plane of symmetry and a number of secondary planes perpendicular to this. The differences between the two systems are due to the difference in the number of these secondary planes and in the number of the lateral axes to which they give rise. We shall see, before the discussion of the tetragonal system is finished, that there is a close analogy between this system and the hexagonal system, and that a familiarity with the forms of the latter will prove a great assistance in obtaining a knowledge of the former.

#### HOLOHEDRAL DIVISION.

(*Ditetragonal Bipyramidal Class.*)

**The Symmetry of the Tetragonal System.**—The complete forms of the tetragonal system are symmetrical with respect to one principal plane of symmetry and four secondary planes of symmetry at right angles to this (Fig. 125). The secondary planes intersect in a common line and are inclined to each other at angles of  $45^\circ$ . In addition to these planes of symmetry there are present in all holohedrons in this system also one axis of fourfold symmetry which coincides with the line of intersection of the secondary planes of symmetry, and four axes of binary symmetry corresponding to the intersections of the secondary planes of symmetry with the principal plane. There is also a center of symmetry (Fig. 126). Figure 127 is a diagrammatic cross section through the most general form of the system, along the plane of the lateral axes.

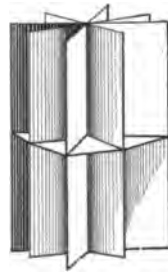


FIG. 125.—Distribution of planes of symmetry in tetragonal holohedrons.



**The Crystallographic Axes.**—The lines chosen as the axes are the lines of intersection of the secondary planes of symmetry and the intersection of alternate secondary planes with the principal

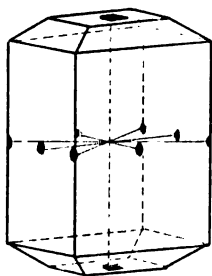


FIG. 126.—Model of symmetry elements in tetragonal holohedrons.

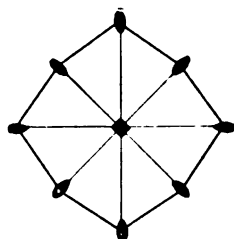


FIG. 127.—Distribution of the elements of symmetry of tetragonal holohedrons projected on the plane of the lateral axes.

plane of symmetry. The axes (Fig. 128) are thus three lines perpendicular to one another at a common point. The two that lie in the principal plane of symmetry are equivalent; i.e., their unities are equal and their ends are similarly terminated. The

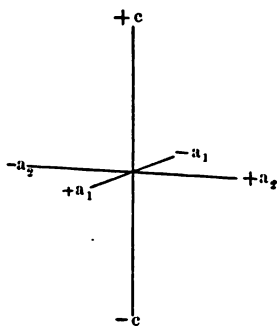


FIG. 128.—System of axes in the tetragonal system.

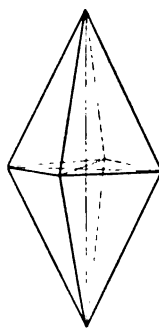


FIG. 129.—Groundform of the tetragonal system. Symbol P, or 111.

third axis is different from the other two—its unity is different from the unity on the other two, and its ends, while terminated alike, are terminated differently from the ends of the lateral axes.

This axis is the vertical axis. It is designated by  $c$ . The two similar axes are the lateral axes. They are designated by  $a$ . The symbol for the system of axes is  $a : a : c$ .

A model representing the axes with their unity lengths would consist of three wires perpendicular to one another at a common point, one shorter or longer than the other two which would be of the same length.

**The Groundform and the Axial Ratio.**—The planes of the groundform in this system must cut the two lateral axes at the same distance from their intersection, and the vertical axis at a greater or less distance than this (Fig. 129). These distances, whatever they may be, are taken as the unity distances on the several axes. The value of the unity on the vertical axis in terms

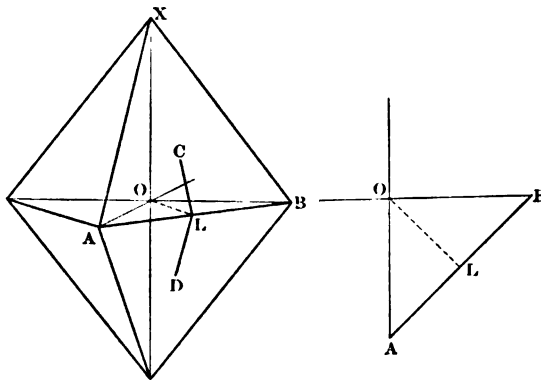


FIG. 130.

of that on the lateral axes is the axial ratio; and this differs for every substance crystallizing in the system. It must, therefore, be calculated for each case, and when once determined it is accepted as representing the ratios between the unities on the vertical and the lateral axes on all crystals of that substance.

The axial ratio for cassiterite ( $\text{SnO}_2$ ) is  $1 : .6724$ . A plane cutting  $c$  at  $.6724$  times the distance at which it cuts  $a$ , cuts both these axes at unity. The plane  $a : a : 2c$  cuts these axes at  $1 : 1 : 1.3448$ .

The groundform in the system consists of eight planes, four above and four below the principal plane of symmetry. These form a bipyramid whose symbol is  $P$  (Fig. 129), and indices 111.

The axial ratio in this system may be determined in a manner exactly analogous to that employed in the hexagonal system (see pp. 57-58). A bipyramid is assumed as the groundform and the angle at its lateral interfacial edge is measured (CLD in Fig. 130). Let one-half of this equal  $\beta$ .

$$OX = OL \tan \beta \quad (1)$$

In the triangle  $OLA$  lying in the plane of the lateral axes

$$\overline{OA}^2 = \overline{OL}^2 + \overline{LA}^2 \quad (2)$$

Since  $OL = LA$ , both being opposite to angles of  $45^\circ$ , and since  $OA = \text{unity on } a$ , equation (2) becomes

$$1 = 2 \overline{OL}^2$$

$$\overline{OL}^2 = 1/2, \text{ and}$$

$$OL = \sqrt{1/2} \text{ substituting in (1) we have}$$

$$OX \text{ or } c = \sqrt{1/2} \tan \beta$$

$$c = .71 \times \tan \beta$$

**The Ditetragonal Bipyramid.**—The most general form in the system is composed of planes that cut the three axes at different distances. Their symbols are  $a : na : mc$ , etc.

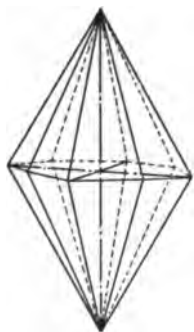


FIG. 131.—Ditetragonal bipyramid,  $mPn$  or  $hkl$ .

When one of the planes is present symmetry demands the presence of fifteen others, and there results a double pyramid consisting of sixteen faces that are scalene triangles (Fig. 131). These meet in two kinds of polar edges and a series of eight lateral edges that are equal. The cross section is an octagon with its alternate angles equal and different from the intervening ones. The lateral solid angles are, therefore, of two kinds, of which the alternate ones are equal.

The lateral axes terminate in the blunter solid angles when  $n > 2.4142$ , and in the more acute ones when  $n < 2.4142$ .

It is crystallographically impossible that all the solid lateral angles should be equal and all the interfacial polar edges similar,

since in this case the value of the parameter  $n$  in the general formula  $a : na : ma$  would be the tan of  $67^{\circ} 30'$  which is  $2.4142 +$ , and this is irrational. For let the figure 132 represent a cross section of the bipyramid through the lateral axes (OA and OD) and let BC and B'C' be the traces of the two planes  $a : na : mc$  and  $na : a : mc$  in the same octant. In the quadrangle OBXB' the angles =  $360^{\circ}$ . The angles  $B + X + B' = 360^{\circ}$ —the angle O which is  $90^{\circ}$ . Consequently the sum of B, X and B' =  $270^{\circ}$  (1). If we assume the interfacial angles to be equal, then  $X = B + B'$ , in which B and B' each equal half of the interfacial angle at

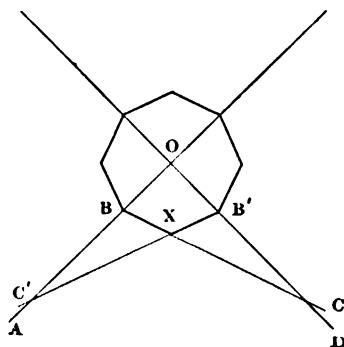


FIG. 132.

the ends of the axes, measured in the plane of the axes, and X the total value of the interfacial angle between these two. Substituting the value of X in (1), we have  $2B + 2B' = 270^{\circ}$ . But B and B' are equal, consequently  $4B = 270^{\circ}$ , or  $B = 67^{\circ} 30'$ . The intercept of the plane represented by BC is unity (OB) on the axis OA. OC (or the intercept on the axis OD) =  $OB \tan 67^{\circ} 30'$ , or since  $OB = 1$ ,  $OC = \text{nat. tan of } 67^{\circ} 30'$  which is  $2.4142 +$ .

The symbol of the ditetragonal bipyramid is  $mPn$ ; its indices are  $hkl$ .

**The Ditetragonal Series.**—A series of forms may be derived from the ditetragonal bipyramid in the same way as the dihex-

agonal series was derived from the dihexagonal bipyramid, by giving different values to the parameter on  $c$ . Thus:

$$(1) a : na : mc$$

$$(3) a : na : c$$

$$(2) a : na : \frac{1}{m}c$$

$$(4) a : na : \infty c$$

$$(5) a : na : \infty c$$

The first three symbols refer to planes of ditetragonal bipyramids that differ from each other merely in the distances at which their planes intercept the  $c$  axis. The fourth symbol represents a plane of the ditetragonal prism  $\infty Pn$  (Fig. 133), and the fifth, the basal plane  $oP$ .

**Series of the First and Second Orders.**—In the symbol  $a : na : mc$ ,  $n$  may be given different values, which will affect the character of the form produced. If  $n$  be given any value

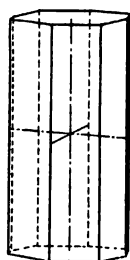


FIG. 133.—Ditetragonal prism,  $\infty Pn$  or  $hko$ .

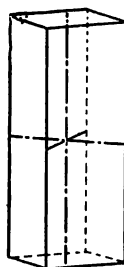


FIG. 134.—Tetragonal prism of the first order,  $\infty P$  or  $110$ .

other than unity or infinity the symbol becomes  $a : n'a : mc$ , which represents a plane of the ditetragonal bipyramid  $mPn'$ .

When  $n = \infty$ , we have  $a : \infty a : mc$ , or  $mP\infty$ , and when  $n = 1$ , we have  $a : a : mc$  or  $P$ . The first is the symbol of a bipyramid of the second order, and the second that of the first order.

#### **Tetragonal Bipyramids and Prisms of the First Order.**—

The symbol  $a : a : mc$  represents a plane cutting the two lateral axes at the same distance, which may be taken as unity, and the  $c$  axis at some distance other than the unity distance on this axis. The form  $mP$ , made up of eight of the planes, which are isosceles

triangles, is a bipyramid of the first order (Fig. 129) in which the lateral axes terminate at the solid angles formed by two planes above and two below the principal plane of symmetry. Its indices are  $hhl$ . By changing the parameter on  $c$  to unity,

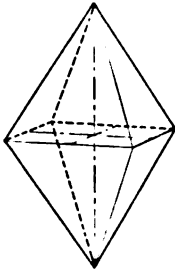


FIG. 135.—Tetragonal bipyramid of the second order,  $mP\infty$  or  $ho$ .

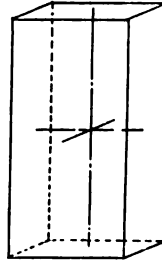


FIG. 136.—Tetragonal prism of the second order,  $\infty P\infty$  or  $100$ .

infinity, and zero, the symbols become  $P$ ,  $\infty P$ , and  $oP$ . The first is the groundform; the second, the prism of the first order (Fig. 134), and the third, the basal plane. The indices of the prism of the first order are  $110$  and of the basal plane  $001$ .

#### Tetragonal Bipyramids and Prisms of the Second Order.

—When  $n$  in the symbol  $a : na : mc$  becomes  $\infty$ , the symbol becomes  $a : \infty a : mc$ . This is a plane of a bipyramid which differs from the bipyramid of the first order only in its position with respect to the axes. These terminate in the centers of the lateral edges, hence, from its analogy with the hexagonal bipyramid of the second order, the form is known as the tetragonal bipyramid of the second order. Its symbol is  $mP\infty$  (Fig. 135). Its indices are  $hol$ .

From this form the prism of the second order is derived in the same way that the prism of the first order is derived from the corresponding pyramid. Symbol,  $\infty P\infty$ . Indices,  $100$ . (Fig. 136.)

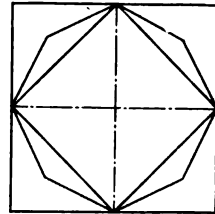


FIG. 137.—Diagram illustrating relations of planes of holohedral pyramids and prisms to axes in tetragonal system.

**Relations between Pyramids and Prisms of the Various Orders.**—The relations existing between the various pyramids and prisms of the tetragonal system are shown in the diagram, figure 137, which is a cross section along the principal plane of symmetry. The inner square gives the position of the bipyramid of the first order with respect to the lateral axes, and the outer

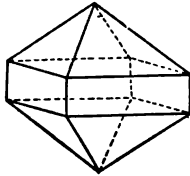


FIG. 138.—Combination of bipyramid and prism of same order.

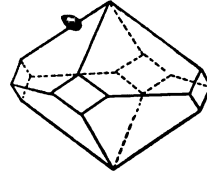


FIG. 139.—Combination of bipyramid and prism of different orders.

square that of the bipyramid of the second order. The octagon between these squares gives the position of the planes of the ditetragonal bipyramid.

**Combinations.**—The combinations of tetragonal forms are similar in character to those of hexagonal forms. When the axial ratio of a mineral is not known, either of its simple tetragonal

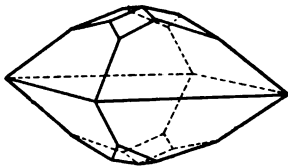


FIG. 140.

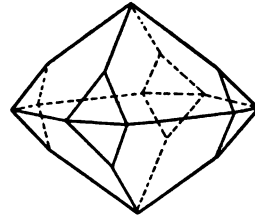


FIG. 141.

Combinations of bipyramids of different orders. The pyramid of the second order has the shorter intercept on  $c$  in Fig. 141 and the longer intercept on this axis in Fig. 142.

prisms or bipyramids may be regarded as the form of the first order, and in this way the position of the axes is fixed. Other simple bipyramids and prisms belong to the series of the first order or to that of the second order according to their relations

with these axes. Ditetragonal bipyramids and prisms are always recognizable by the number of their planes.

Figure 138 is a combination of a bipyramid and prism of the same order, and figure 139 that of a bipyramid and a prism of different orders. Figures 140 and 141 are combinations of pyramids of the two orders. In the former the pyramid of the second order has a shorter parameter on  $c$  than that of the first order, and in figure 141 it has a longer parameter on this axis. Figure 142 is a crystal of anatase ( $\text{TiO}_2$ ) with  $1/3 P(z)$ ,  $1/7 P(v)$ ,  $\infty P(m)$ ,  $\infty P \infty(a)$ , and  $P \infty(e)$ .

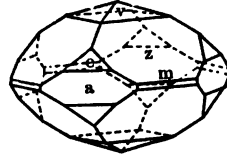


FIG. 142.—Crystal of anatase containing  $1/3 P(z)$ ,  $1/7 P(v)$ ,  $\infty P(m)$ ,  $\infty P \infty(a)$  and  $P \infty(e)$ .

### HEMIHEDRAL DIVISION.

**Hemihedrism in the Tetragonal System.**—Although the hemihedral division of this system is by no means as important as the hemihedral division of the hexagonal system, it nevertheless deserves consideration since several well-known minerals exhibit forms belonging to it.

**Possible Kinds of Hemihedrism.**—In the tetragonal system there are three classes of hemihedrons distinguished by differences in symmetry. They may be regarded as derived from the holohedrons by the three methods indicated in the subjoined figures.

The first method (Fig. 143) is known as sphenoidal hemihedrism; the second, pyramidal hemihedrism (Fig. 144), and the third, trapezohedral hemihedrism (Fig. 145).

Forms derived by the third method have not been observed on minerals. Those derived by the second method are found on several rare minerals. The sphenoidal forms are seen on the common mineral chalcopyrite ( $\text{CuFeS}_2$ ).

**The Sphenoidal Hemihedrons** (*Tetragonal Scalenohedral Class*).—This class of hemihedrons may be considered as being derived from the holohedrons by the suppression of the planes in alternate octants and the extension of all others. This is



analogous to tetrahedral hemihedrism in the isometric system, and rhombohedral hemihedrism in the hexagonal system.

In this class of hemihedrons the principal plane of symmetry and the two alternate secondary planes passing through the

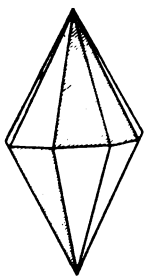


FIG. 143.

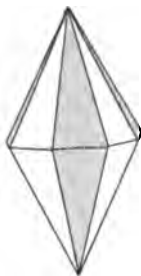


FIG. 144.

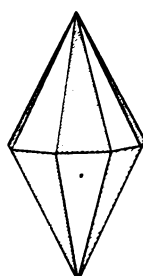


FIG. 145.

Figures illustrating the possible methods of derivation of the hemihedrons in the tetragonal system. Fig. 143, sphenoidal; Fig. 144, pyramidal; Fig. 145, trapezohedral.

lateral axes disappear, as do also two of the axes of binary symmetry and the center of symmetry. Moreover, the fourfold axis is changed to a binary axis. The remaining elements of symmetry are three binary axes of symmetry perpendicular to one another and coinciding with the crystallographic axes, and the two secondary planes of symmetry passing between the lateral crystal axes (Fig. 146). Four new geometrical forms belong to this class, two derived from the ditetragonal bipyramid and the other two from the bipyramid of the first order.

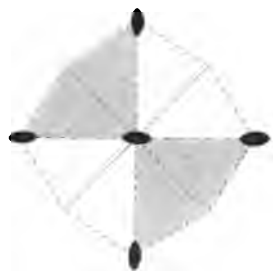


FIG. 146.—Diagram illustrating distribution of symmetry elements in sphenoidal hemihedrons.

*Tetragonal Scalenohedrons.*—The most general form of the system,  $mPn$ , gives rise to two congruent scalenohedrons, each composed of 8 similar scalene triangles meeting in three kinds of interfacial angles and each possessing two kinds of solid angles. The two planes passing through the edges in which the vertical axis terminates are perpendicular to

each other. The lateral axes terminate in the centers of the edges which are not in these planes.

The symbols of the forms are  $+\frac{mPn}{2}$  (Fig. 147) and  $-\frac{mPn}{2}$  (Fig. 148), or  $\kappa(hkl)$  and  $\kappa(h\bar{k}l)$ .

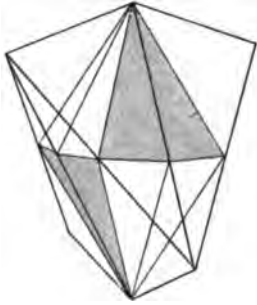


FIG. 147.—Positive tetragonal scalenohedron,  $+\frac{mPn}{2}$  or  $\kappa(hkl)$ .

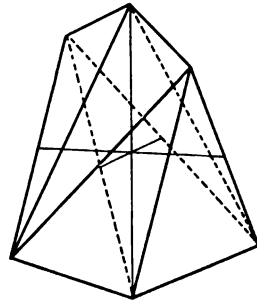


FIG. 148.—Negative tetragonal scalenohedron,  $-\frac{mPn}{2}$  or  $\kappa(h\bar{k}l)$ .

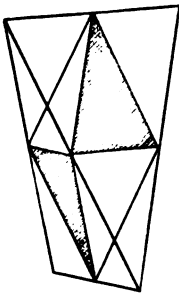


FIG. 149.—Positive tetragonal sphenoid,  $+\frac{mP}{2}$  or  $\kappa(hhl)$ .

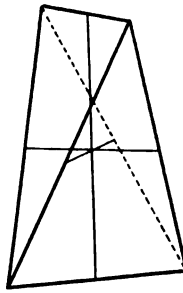


FIG. 150.—Negative tetragonal sphenoid,  $-\frac{mP}{2}$ , or  $\kappa(h\bar{h}l)$ .

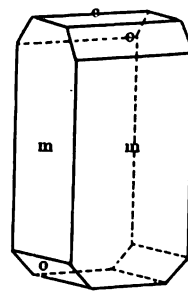


FIG. 151.—Crystal of urea with  $oP(c)$ ,  $\infty P(m)$  and  $+\frac{P}{2}(o)$ .

**Tetragonal Sphenoids.**—The tetragonal sphenoids contain half the planes of the bipyramid of the first order. They differ in appearance from the isometric tetrahedron in that they are not of equal dimensions along the three axes. The forms consist

of four similar isoscles triangles meeting in two kinds of interfacial edges.

Their symbols are  $+\frac{mP}{2}$  (Fig. 149) and  $-\frac{mP}{2}$  (Fig. 150), or  $\kappa(hhl)$  and  $\kappa(h\bar{h}l)$ . Figure 151 shows the combination of  $oP(c)$ ,  $\infty P(m)$ , and  $+\frac{P}{2}(o)$ .

**The Pyramidal Hemihedrons** (*Tetragonal Bipyramidal Class*).—The hemihedrons of this class may be derived from the

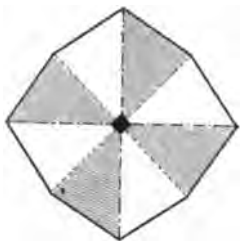


FIG. 152.—Diagram illustrating distribution of symmetry elements in pyramidal hemihedrons.

holohedrons by the extension of all the planes lying in the alternate sections into which the secondary planes of symmetry divide them.

These forms retain the principal plane of symmetry, the axis of fourfold symmetry, and the center of symmetry. The secondary planes and binary axes of symmetry found in the holohedrons disappear (Fig. 152).

There are four new forms in this class, of which two are derived from the ditetragonal bipyramid and two from the corresponding prism. The forms are congruent. They correspond to the pyramidal hemihedrons of the hexagonal system and the parallel hemihedrons of the isometric system.

*Tetragonal Bipyramids and Prisms of the Third Order.*—These are forms that resemble geometrically the bipyramids and prisms of the first and second orders. They differ from them, however, in the relation of their planes to the crystal axes.

The pyramids of the third order are derived from the ditetragonal bipyramids, hence their Naumann symbols are  $+\left[\frac{mPn}{2}\right]$  and  $-\left[\frac{mPn}{2}\right]$ , the brackets indicating that they belong to the pyramidal class. The prisms of the third order are derived from the ditetragonal prisms, and are represented by the symbols  $+\left[\frac{\infty Pn}{2}\right]$  and  $-\left[\frac{\infty Pn}{2}\right]$ . Their corresponding indices

are  $\pi(hkl)$  and  $\pi(h\bar{k}l)$ , and  $\pi(hko)$  and  $\pi(h\bar{k}o)$ . The relations of the planes of the positive forms to those of the pyramids and prisms of the first, second, and ditetragonal orders are shown in figure 153. The heavy lines indicate the positions of the planes belonging to the forms of the third order.

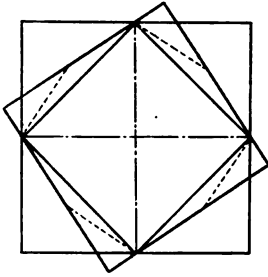


FIG. 153.—Diagram illustrating relations of planes of pyramids and prisms of various orders to the axes in the tetragonal system.

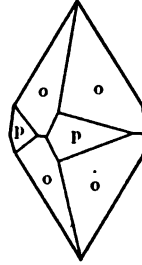


FIG. 154.  
Crystal of stolzite with  $P(o)$  and  $[\infty P_{2/3}^{4/3}](p)$ .

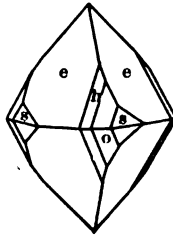


FIG. 155.—Crystal of scheelite with  $P\infty(e)$ ,  $P(o)$ , and  $[P_2^3](h)$  and  $[3P_2^3](s)$ .

**Combinations.**—In combination the pyramids and prisms of the third order are distinguished from those of the first and second orders by their positions on the crystal. Figure 154 represents a crystal of stolzite ( $PbWO_4$ ) bounded by  $P(o)$  and

+  $\left[ \frac{\infty P_4/3}{2} \right] (p)$ , and figure 155, one of scheelite ( $\text{CaWO}_4$ ) on which occur  $P \infty (e)$ ,  $P (o)$ , +  $\left[ \frac{P_3}{2} \right] (h)$  and -  $\left[ \frac{3P_3}{2} \right] (s)$ .

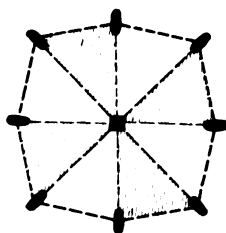


FIG. 156.—Diagram illustrating distribution of symmetry elements in trapezohedral hemihedrons.

**The Trapezohedral Hemihedrons** (*Tetragonal Trapezohedral Class*).—By the extension of the alternate planes on the dihexagonal bipyramid two new forms are derived, and these are

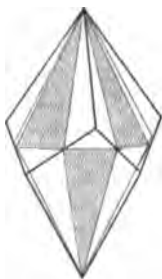


FIG. 157.—Right tetragonal trapezohedron,  $r \frac{mP_n}{2}$ , or  $\tau(hkl)$ .

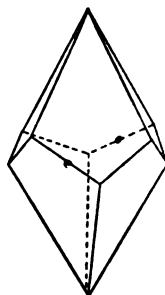


FIG. 158.—Left tetragonal trapezohedron,  $l \frac{mP_n}{2}$ , or  $\tau(h\bar{k}l)$ .

enantiomorphous. They retain the axis of fourfold symmetry and the four axes of binary symmetry, but have lost all planes of symmetry and the center of symmetry (Fig. 156). The forms are known as the right and left tetragonal trapezohedrons and

are given the symbols  $r \frac{mPn}{2}$  (Fig. 157),  $l \frac{mPn}{2}$  (Fig. 158) or  $\tau(hkl)$  and  $\tau(h\bar{k}l)$ . They are characterized by having four equal polar edges and a zig-zag lateral edge composed of long and short courses. The lateral axes terminate in the centers of the larger zig-zag edges. Because they have not yet been found on crystals they are not discussed in detail.

**TABULAR LIST OF HEMIHEDRONS IN THE TETRAGONAL SYSTEM.**

(See explanation under list of isometric hemihedrons, p. 51.)

Holohedrons	Hemihedrons		
	Sphenoidal	Pyramidal	Trapezohedral
$mPn$	$\pm \frac{mPn}{2}$	$\pm \left[ \frac{mPn}{2} \right]$	r.l. $\frac{mPn}{2}$
$mP\infty$	$mP\infty$	$mP\infty$	$mP\infty$
$\infty Pn$	$\infty Pn$	$\pm \left[ \frac{\infty Pn}{2} \right]$	$\infty Pn$
$\infty P\infty$	$\infty P\infty$	$\infty P\infty$	$\infty P\infty$
$mP$	$\pm \frac{mP}{2}$	$mP$	$mP$

**Hemimorphism.**—Hemimorphism is observed in the following classes of the tetragonal system, viz., holohedrons and pyramidal hemihedrons.

**Tetartohedrism.**—Although tetartohedral forms are possible in this system, no crystals have been observed with tetartohedrons upon them, consequently they are not discussed.

## CHAPTER IX.

### THE ORTHORHOMBIC SYSTEM.

#### **Systems Possessing No Principal Plane of Symmetry.—**

As the hexagonal and the tetragonal systems are classed together in consequence of the possession by them of one principal plane of symmetry, so the orthorhombic, the monoclinic, and the triclinic systems may be united into a group characterized by the entire lack of principal planes of symmetry. These systems possess certain analogies which are expressed in part by the names given to their characteristic forms. In each system there are three axes, and these all possess different unities. No two are equivalent, hence the symbol of the axes for each system is  $a : b : c$ .

**The Orthorhombic System.**—The holohedral division of the orthorhombic system includes forms possessing three secondary planes of symmetry. These are perpendicular to each other, and divide space into eight octants (Fig. 159). Its forms possess in addition three axes of binary symmetry, perpendicular to the three planes of symmetry, and a center of symmetry (Figs. 160 and 161).

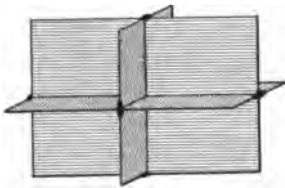


FIG. 159.—Distribution of planes of symmetry in orthorhombic holohedrons.

**Axes of the System.**—The lines chosen as the axes of the system are the lines of intersection of the three planes of symmetry in the holohedrons. They are consequently three lines at right angles to each other at a common point (Fig. 162). Since there is no plane of symmetry situated between any two of the axes, no two of them can be equivalent, hence no two can possess the same unities. The two ends of each axis, however, are equivalent since they are separated in each case by one of the planes of symmetry, hence, except in cases of hemimorphism, the two ends of each

axis must be similarly terminated. Because of the center of symmetry, the forms must have parallel sides.

**Designation of the Axes.**—Since there is no principal plane of symmetry, there is no one axis that differs from the other axes in any essential particular. Any one of the axes may be selected as the vertical axis, when the other two become the lateral axes. One of these is longer than the other. The longer axis is (Fig. 162) designated as the *macroaxis*, and the shorter as the *brachyaxis*.

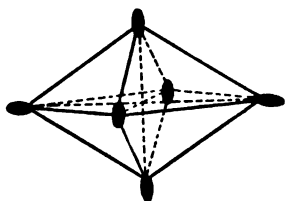


FIG. 160.—Model showing distribution of symmetry elements in orthorhombic holohedrons.

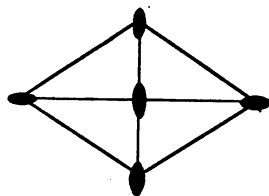


FIG. 161.—Symmetry elements of orthorhombic holohedrons, projected on the plane of the lateral axes.

In the scheme of the axes, the brachyaxis runs from front to back—is the  $a$  axis; the macroaxis from left to right—is the  $b$  axis; and the vertical axis from below to above—is the  $c$  axis.

In the symbols representing the axes and the crystal planes, the sign  $\sim$  always refers to the brachyaxis, and the sign  $-$  to the macroaxis. The symbol of the axes thus becomes  $\tilde{a} : \tilde{b} : c'$ .

**Groundform and Axial Ratio.**—The groundform in this system is composed of planes cutting the three axes at different distances, which are assumed as the unity (Fig. 163) distances on the several axes. The entire form consists of eight planes, one in each octant, constituting a bipyramid whose faces are unequilateral triangles. In cross section the pyramid is a rhomb (Fig. 164) and not a square (Fig. 165) as in the case of the tetragonal pyramid.

A different groundform is chosen for each substance crystallizing in the system. From this the relative lengths of the unities on the  $c$  and the  $\tilde{a}$  axes are determined in terms of the unity on  $\tilde{b}$ , and these are taken as the unities to which all of



the planes occurring on the crystals of this substance are referred. The relative values of the unities on the three axes constitute the axial ratio, which in this system contains one more term than the axial ratio in the tetragonal and in the hexagonal systems.

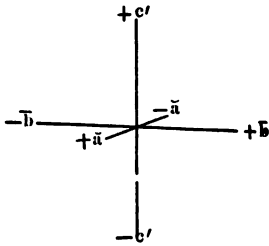


FIG. 162.—Axes of the orthorhombic system.

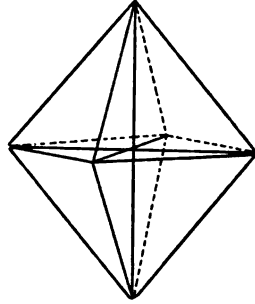


FIG. 163.—Groundform in the orthorhombic system. P or III.

The value of the axial ratio is so characteristic for each mineral that it is always given under the name of the mineral in the larger text-books on mineralogy. The axial ratio for olivine  $[(\text{MgFe})_2\text{SiO}_4]$ , for instance, is  $\bar{a} : \bar{b} : c = .4657 : 1 : .5865$ . This means that the plane which cuts the three axes at the unities

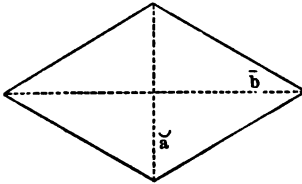


FIG. 164.

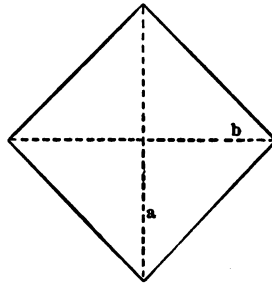


FIG. 165.

Cross sections of groundforms in orthorhombic (Fig. 164) and tetragonal (Fig. 165) systems.

intercepts the  $a$  axis at .4657 times the distance from the intersection of the axes as that at which it cuts the  $b$  axis; and the  $c$  axis at .5865 times this distance. The plane that cuts the three axes at .9314 : 1 : 1.7595, respectively, is the plane  $2\bar{a} : \bar{b} : 3c$ .

## HOLOHEDRAL DIVISION.

(Orthorhombic Bipyramidal Class.)

**The Most General Form.**—The most general form in the system is composed of planes cutting the three axes at different distances, which do not bear to each other the relations of the unity distances. The symbol of one of its planes may be either  $\bar{n}\bar{a} : b : mc$  or  $\bar{a} : n\bar{b} : mc$ . These symbols do not represent different planes on the same form, as corresponding symbols do in the tetragonal system, because symmetry does not demand the presence of both, when either is present. The planes occur independently. The plane  $\bar{n}\bar{a} : \bar{b} : mc$  requires the presence of  $\bar{n}\bar{a} : -b : mc$ , and also  $-\bar{n}\bar{a} : b : mc$  and  $-\bar{n}\bar{a} : -\bar{b} : mc$ , above the horizontal plane of symmetry, and the presence of four corresponding planes below the symmetry plane; but none of these planes demands the presence of a plane cutting the axis  $\bar{b}$  at  $n\bar{b}$ .

The plane  $\bar{a} : nb : mc$  likewise demands the presence of seven more similar planes, but its presence on a crystal does not demand the presence of any plane cutting the  $\bar{a}$  axis at  $n$ . Consequently the most *general form* in the orthorhombic system is an *eight-faced bipyramid*.

**Three Series of Forms.**—As in the tetragonal system there are three series of pyramids and prisms in the orthorhombic system. These are named with reference to their relations to the lateral axes. *It is customary to regard the shorter distance at which a plane cuts the two lateral axes unity* and to call the series by the name of the axis upon which the parameter is not unity, provided the two lateral axes are not both cut at unity.

There are thus three series of forms—the *groundform* or *unit series*, the *brachy series*, and the *macro series*.

**The Unit Pyramids and Prisms.**—The *unit pyramids* and *unit prisms* are composed of planes cutting the two lateral axes at unity. These two axes are cut at different distances, but they are distances that bear to each other the same ratio as do the unities on the axes. The intercept on the vertical axis may be unity or any multiple or small fraction of this.

The unit pyramids, called also the orthorhombic bipyramids,

are composed of 8 unequilateral triangles whose apices are at the terminations of the vertical axis. A cross section through the lateral axes would exhibit a rhomb, whose diagonals would represent these axes. The symbol of one of its planes is  $\bar{a} : \bar{b} : mc$ , and the symbol of the form is  $P$  (111) or  $mP$  ( $hhl$ ).

The groundform is that member of this series of pyramids in which the intercept on the  $c$  axis is unity. For instance, if the axial ratio of a given substance is  $.4657 : 1 : .5865$ , and its crystals contain planes whose intercepts on the  $\bar{a}$  and  $\bar{b}$  axes bear the relation  $.4657 : 1$ , these planes belong to the unit series of pyramids. If the relation of the intercepts on the  $\bar{b}$  and  $c$  axes

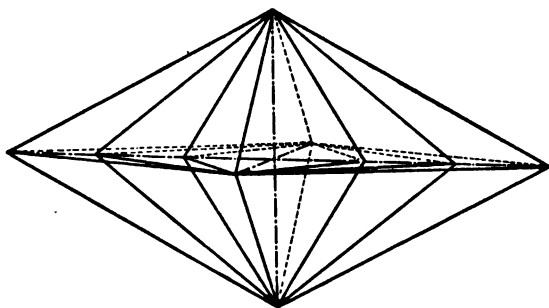


FIG. 166.—Groundform and two macro pyramids in the orthorhombic system,  $P$ ,  $Pn$  and  $Pn'$ .

is as  $1 : .5865$  the pyramid is  $P$ , or the groundform. If the ratio between the intercepts on  $\bar{b}$  and  $c$  are as  $1 : 1.7595$ , the symbol of the form is  $3P$  (i.e.,  $mP$ ).

When  $m$  becomes  $\infty$  the planes of the form are all parallel to  $c$ , and there results the unit prism whose symbol is  $\infty P$  or  $110$ . This form differs from the corresponding tetragonal prism in its cross section.

**The Macroseries.**—This series consists of forms whose planes cut the brachyaxis ( $\bar{a}$ ) at unity and the macroaxis ( $\bar{b}$ ) at some distance other than unity.

The most general symbol of a plane belonging to this series is  $\bar{a} : m\bar{b} : mc$ . The form composed of planes of this kind is a *macro-bipyramid* which can be distinguished from the unit *bipyramid* only when the unity on  $b$  is known. Its symbol is  $mPn$  or ( $hkl$ ).

The macro-mark over the  $n$  signifies that this parameter refers to the macroaxis. Fig. 166 represents the groundform and two macropyrramids with different intercepts on  $\bar{b}$ .

It sometimes happens that two crystallographers working on the same crystal choose different pyramids for the groundform and thus obtain different axial ratios for the same substance. But in these cases the two groundforms chosen may bear to each other the relations of unit pyramids to macropyrramids or brachypyrramids. In the example of olivine cited above (p. 104) the axial ratio accepted is  $.4657 : 1 : .5865$ . The groundform  $P$  cuts the three axes at the relative distances indicated. The plane  $\bar{a} : 2\bar{b} : c$ , cuts these axes at  $.4657 : 2 : .5865$ . It is easily conceivable that some crystallographer might prefer to use as the groundform the pyramid composed of planes cutting the three axes at these distances. If so, his choice of axial ratios would be:  $.23285 : 1 : .29325$ . The original groundform would then be a brachypyrmaid,  $2\bar{a} : b : 2c$  or  $2P\bar{2}$ . Thus in order to distinguish between macropyrramids, brachypyrramids, and unit pyramids on crystals it is necessary to know what axial ratio is accepted by crystallographers as their standard of reference for its forms.

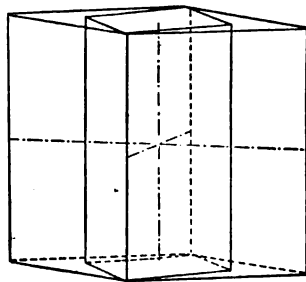


FIG. 167.—The unit prism and a macropism in the orthorhombic system,  $\infty P (110)$  and  $\infty P\bar{n} (hko)$ .

When  $m$  in the symbol  $mPn$  becomes unity a *macropyrmaid* results that differs from  $mPn$  in the inclination of its faces to the  $c$  axis. Its symbol is  $Pn$ , or  $(hlh)$ .

When  $m$  becomes  $\infty$  we have the form  $\infty Pn$ , or  $(hko)$  which is the *macropism*. Figure 167 shows the relation between the unit and one of the macropisms.

**The Brachyseries.**—In addition to the macroseries there is also in this system a series of brachypyrramids (Fig. 168) and prisms, which differs from the macroseries in the fact that the lateral parameter which is not unity applies to the brachyaxis. The symbols of the brachyseries are distinguished from those

of the macroseries by the use of the mark  $\sim$  over the parameter that refers to the brachyaxis.  $P\tilde{n}$ ,  $mP\tilde{n}$ ,  $(khl)$ , etc., are *brachypyramids*, while  $\infty P\tilde{n}$  ( $kho$ ) is the *brachyprism*.

**Combinations.**—In combination the forms of the unit series, the macroseries, and the brachyseries are not difficult to distinguish when they occur together, provided the unit forms can be recognized. The macroforms occur at the terminations of the brachyaxis and the brachyforms at the termination of the macroaxis. When the forms of either series occur alone, how-

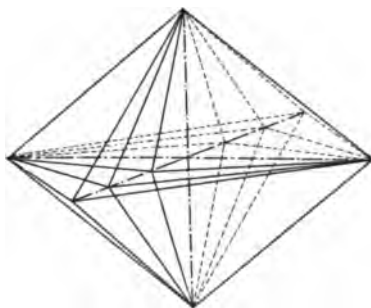


FIG. 168.—Groundform and two brachybipyramids in the orthorhombic system,  $P$ ,  $P\tilde{n}$  and  $P\tilde{n}'$ .

ever, they can be recognized only by the measurement of their interfacial angles and the calculation from these of the parameters on the lateral axes (see Fig. 174).

**Domes.**—There are other classes of prisms in this system that have been given the distinctive name domes. These consist of planes that are parallel to one of the lateral axes, while they intercept the other lateral axis and the vertical axis at certain definite distances. When parallel to the macroaxis, the forms produced by them are called *macrodomes* (Fig. 169); when parallel to the brachyaxis, the forms are the *brachydomes* (Fig. 170).

In writing the symbols of the domes the parameter on the lateral axis which is not  $\infty$  is made unity. Thus the symbols of planes of the macrodomes are  $\tilde{a} : \infty \tilde{b} : c$ , or  $\tilde{a} : \infty \tilde{b} : mc$ , according as the  $c$  axis is cut at unity or at some other distance. When  $m$  becomes  $\infty$ , the plane is called a *pinacoid*. (See next section.)

The symbols of the macrodomes are  $P\infty$ ,  $mP\infty$ , (*hol*), and of the brachydomes,  $P\infty$ ,  $mP\infty$ , (*ohl*). Each dome consists of four faces, uniting in pairs at the terminations of the vertical axis. The macrodomes are in the angles between the vertical and the brachyaxis (Fig. 169), and the brachydomes in the angles between the vertical and the macroaxis (Fig. 170).

**Pinacoids.**—The pinacoids embrace those forms with planes parallel to two axes at the same time. Each form consists of a pair of parallel planes. When these are parallel to the two lateral

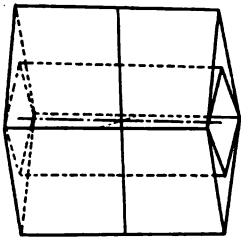


FIG. 169.—Two orthorhombic macrodomes,  $P\infty$  and  $mP\infty$ , 101 and *hol*.

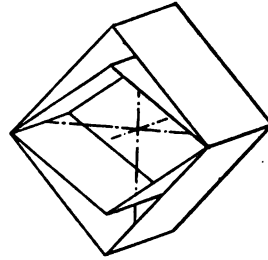


FIG. 170.—Two orthorhombic brachydomes,  $P\infty$  and  $mP\infty$ , or 011 and *ohl*.

axes they constitute the *basal plane* or the *basal pinacoid*, similar to the corresponding form in the tetragonal system. Its symbol is written  $oP$  or (*001*).

When the planes of the form are parallel to the vertical and the macroaxis, the form is termed the *macropinacoid*, and when parallel to the vertical and the brachyaxis it is called the *brachypinacoid*. The symbol of the former is  $\infty P\infty$  (100), and of the latter  $\infty P\infty$ , or 010 (Fig. 171).

**Closed Forms.**—It will be noted that the only forms that will completely enclose space in the orthorhombic system are the bipyramids. *No prism, pinacoid, or dome will alone enclose space.* All are open forms. Consequently any crystal that contains on it a plane belonging to any one of these three forms cannot be represented correctly by less than two symbols, no matter how simple the habit of the crystal.

**Combinations.**—From a consideration of the statements

made in the preceding paragraph, it is plain that the number of forms present in combination on orthorhombic crystals is usually larger than the number found on tetragonal crystals. Figure 172 represents a combination of  $\infty P(p)$ ,  $\infty P\bar{\infty}(b)$ ,  $oP(c)$ , and  $2P\bar{\infty}(q)$ , while figure 173 illustrates a more complicated combination of  $\infty P(p)$ ,  $\infty P\bar{2}(p')$ ,  $oP(c)$ ,  $P\bar{\infty}(q)$ ,  $P(o)$ ,  $1/2P(o')$ ,  $1/3P(o'')$ , and  $2/3P\bar{2}(x)$ . Figure 174 represents a combination sometimes seen on andalusite ( $Al_2SiO_5$ ),  $\infty P\bar{\infty}(a)$ ,  $\infty P\infty(b)$ ,  $oP(c)$ ,  $\infty P(m)$ ,  $\infty P\bar{2}(l)$ ,  $\infty P\bar{2}(n)$ ,  $P\infty(r)$ ,  $P(p)$ ,  $P\bar{\infty}(s)$ , and  $2P\bar{2}(k)$ . A very complex

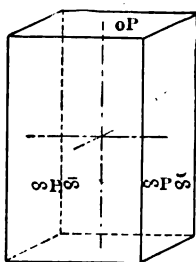


FIG. 171.—Combination of orthorhombic pinacoids and basal plane.

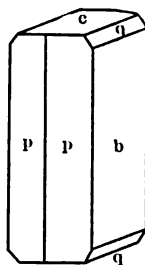


FIG. 172.—Crystal of olivine with  $\infty P(p)$ ,  $\infty P\bar{\infty}(b)$ ,  $oP(c)$  and  $2P\bar{\infty}(q)$ .

crystal of olivine  $[(FeMg) SiO_4]$  is shown in Fig. 175. It contains the forms  $\infty P\bar{\infty}(a)$ ,  $oP(c)$ ,  $\infty P(n)$ ,  $\infty P\bar{2}(s)$ ,  $\infty P\bar{3}(r)$ ,  $P\bar{\infty}(d)$ ,  $P\bar{\infty}(h)$ ,  $2P\bar{\infty}(k)$ ,  $4P\bar{\infty}(i)$ ,  $P(e)$ ,  $1/2P(o)$ ,  $2P\bar{2}(f)$ , and  $3P\bar{3}(l)$ .

Orthorhombic crystals are frequently elongated in the direction of one axis. This direction is usually taken as the vertical axis of the crystals, and the latter are said to possess a prismatic, a columnar, or an acicular habit (Fig. 172). When disproportionately shortened in the direction of a single axis, this axis is likewise often regarded as a vertical axis, and the crystal is said to be tabular in habit.

**Hemimorphism** (*Orthorhombic Pyramidal Class*).—The hemimorphic development of forms in this system is of considerable importance. With the disappearance of the planes of the holohedrons from one end of an axis there is necessarily the disap-

pearance of one of the planes of symmetry, the two axes of symmetry lying in this plane, and the center of symmetry. The elements of symmetry remaining are two planes of symmetry and the axis of binary symmetry coinciding with their intersection.

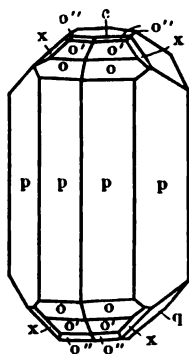


FIG. 173.—Crystal of topaz. See text for symbols of planes.

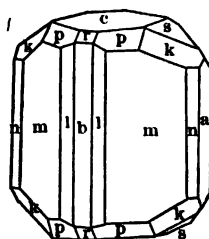


FIG. 174.—Crystal of andalusite. See text for symbols of planes.

This axis is necessarily polar, because of the absence of the plane of symmetry perpendicular to it. It is usually made the vertical axis  $c$ .

All forms except those parallel to two axes, i.e., the prism, the

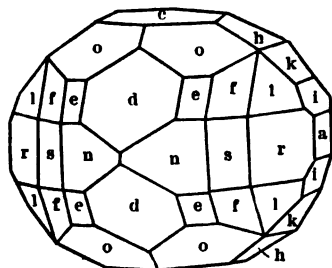


FIG. 175.—Crystal of olivine. See text for symbols of planes.

macropinacoid, and the brachypinacoids, now separate into upper and lower halves, either of which may occur alone.

Figure 176 represents a crystal of struvite ( $\text{NH}_4\text{MgPO}_4 + 6\text{H}_2\text{O}$ ) containing  $\infty P \tilde{\infty}(b)$ ; at the upper pole,  $\frac{P \tilde{\infty}}{2} u(r)$ ,  $\frac{P \tilde{\infty}}{2} u(q)$ ,



$\frac{4P_{\infty}}{2} u (q')$ ; and at the lower pole,  $\frac{oP}{2} l (c)$  and  $\frac{1/3P_{\infty}}{2} l (r')$ .

Figure 177 is a crystal of calamine ( $Zn_2(OH)_2SiO_3$ ) with  $\infty P_{\infty} (b)$ ,  $\infty P_{\infty} (a)$ ,  $\infty P (p)$ ; at the upper pole,  $\frac{3P_{\infty}}{2} u (r')$ ,  $\frac{P_{\infty}}{2} u (r)$ ,  $\frac{3P_{\infty}}{2} u (q')$ ,  $\frac{P_{\infty}}{2} u (q)$ ,  $\frac{oP}{2} u (c)$ ; and at the lower pole,  $\frac{2P_2}{2} l (o)$ .

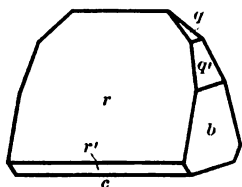


FIG. 176.—Hemimorphic crystal of struvite. See text for symbols of planes.

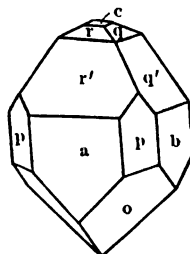


FIG. 177.—Hemimorphic crystal of calamine. See text for symbols of planes.

### HEMIHEDRAL DIVISION.

**Kinds of Hemihedrism.**—The only forms in the orthorhombic system that can yield new hemihedrons are the pyramids. One-half the faces of these forms may be extended in three different ways, but only in one case will new forms be derived which will comply with the demand of hemihedrism. Consequently there is in this system but *one kind of hemihedrism*, and by it only *one new type of hemihedral form* is produced.

#### **Sphenoidal Hemihedrism** (*Orthorhombic Bisphenoidal Class*).

—The only hemihedrons in this system are two sphenoids which may be regarded as derived from the various pyramids by the extension of planes in alternate octants. These new forms possess only three axes of binary symmetry. The three planes of symmetry and the center of symmetry have disappeared (Fig. 178).

This type of hemihedrism is analogous to the inclined hemihedrism of the isometric system and the sphenoidal hemihedrism of the tetragonal system.

*Sphenoids.*—The sphenoids are four-sided closed figures composed of four scalene triangles meeting in six interfacial edges, two of which are equal and are different from the other four which are also equal. The vertical axis terminates in the centers of the two equal edges, and the lateral axes in the centers of the other



FIG. 178.—Diagram illustrating distribution of symmetry elements in orthorhombic sphenoidal hemihedrons.

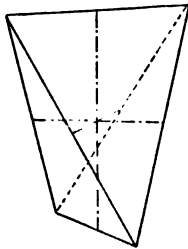


FIG. 179.—Right orthorhombic bisphenoid  $r \frac{mP}{2}$ , or  $hhl$ .

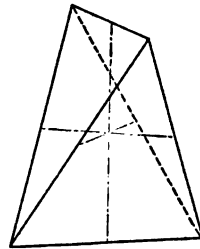


FIG. 180.—Left orthorhombic bisphenoid,  $- \frac{mP}{2}$ , or  $hhl$ .

four. The form differs in appearance from the tetragonal sphenoid from the fact that the edges at the terminations of the vertical axes are not at right angles. The difference between the value of this angle and  $90^\circ$  increases with the difference in the lengths of the two lateral axes.

From every pyramid two of these forms are produced. They are *enantiomorphous*, and are therefore known as right (Fig. 179)

and left forms (Fig. 180). Their symbols are  $r\frac{mP}{2}(hhl)$ ,  $l\frac{mP}{2}$ ,  $hhl$ ,  $r\frac{mP\bar{n}}{2}(khl)$ ,  $l\frac{mP\bar{n}}{2}(h\bar{k}l)$ ,  $r\frac{mP\bar{n}}{2}(hkl)$ , and  $l\frac{mP\bar{n}}{2}(khl)$ .

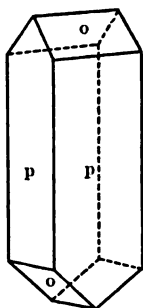


FIG. 181.—Crystal of epsom salts with  $\infty P(p)$  and  $r_2^P(o)$ .

A combination of  $\infty P(p)$  and  $r_2^P(o)$  as seen on crystals of epsom salts ( $MgSO_4 + 7H_2O$ ) is illustrated in Fig. 181.

## CHAPTER X.

### THE MONOCLINIC SYSTEM.

#### **Symmetry of the Holohedrons of the Monoclinic System.**

—Complete forms belonging to the monoclinic system possess but a single plane of symmetry. Consequently monoclinic holohedrons are bilaterally symmetrical; i.e., they possess two sides that are alike. They possess also an axis of binary symmetry perpendicular to the plane of symmetry and a center of symmetry (Fig. 182). Figure 183 is a diagrammatic representation of the distribution of the elements of symmetry in the plane of the lateral axes.

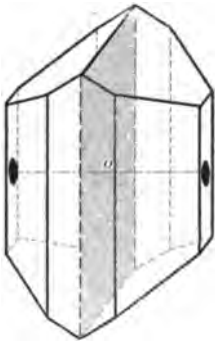


FIG. 182.—Model of monoclinic crystal showing distribution of elements of symmetry in holohedrons.

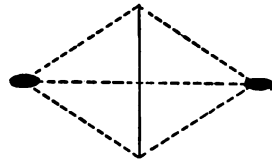


FIG 183.—Diagram illustrating distribution of symmetry elements of monoclinic holohedrons.

**Axes.**—The single plane of symmetry in this system determines the position of one line which may be chosen as one of the axes of reference for the planes of monoclinic forms. This line is the axis of symmetry which is perpendicular to the plane of symmetry. The other two axes must necessarily lie in the plane of symmetry, but their directions in this plane are a matter of choice, to be decided in the case of each substance as may be most convenient.

The two axes that lie in the plane of symmetry are at right angles to the third axis, but are inclined to each other at some angle

other than  $90^\circ$ . Their angle of inclination is always designated as the angle  $\beta$  (Fig. 184).

Symmetry demands that the two terminations of the axis that is normal to the plane of symmetry shall be equivalent in all respects. *There is, however, nothing in the symmetry of the system which necessitates the equivalency of any two of the three axes or of the opposite ends of the axes in the plane of symmetry.*

Hence the unities on the three axes are different in value.

A model representing the axes of the monoclinic system would be composed of three lines of unequal lengths intersecting at a common point. Two of these must necessarily be inclined to each other at some angle other than  $90^\circ$ , and the third must be normal to the plane of these.

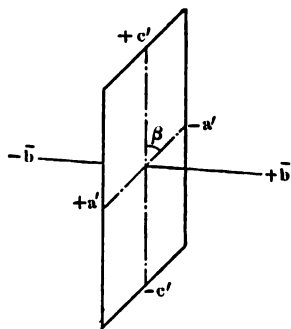


FIG. 184.—Relation of axes to plane of symmetry in monoclinic holohedrons.

**Designation of the Axes.**—It is customary in studying monoclinic crystals to place them in such a position that the plane of symmetry shall stand vertically. The line normal to this plane then takes the position of the  $b$  axis. Of the other two axes, one is made vertical and the other is so placed that it inclines toward the observer. The acute angle  $\beta$  is thus on the back of the crystal (Fig. 184).

As in the other systems, the axis that stands vertically is the axis  $c$  or the *vertical axis*. The axis corresponding to the  $b$  axis of the orthorhombic system, i.e., the one that is normal to the plane of symmetry, is the *orthoaxis*. The third axis—the one inclined to the vertical axis—is the *a* axis. It is known as the *clinoaxis*. The sign used in symbols to designate reference to the  $b$  axis is the same as that which designates reference to the macroaxis in the orthorhombic system. The sign used to designate the clinoaxis is  $\backslash$ . The symbol for the axes is thus  $a : b : c$ .

**Groundform and Crystallographic Constants.**—As in the

orthorhombic system, so in the monoclinic system, any form composed of planes cutting the three axes at finite distances may be assumed as the groundform, and from it the lengths of the unities on the three axes may be determined. These unities are expressed in terms of the unity on  $b$  as 1, and when once determined for a crystal of any substance this ratio is accepted as the axial ratio for all crystals of that substance.

In addition to the axial ratio there is one other determination necessary to fix the position of the axes in this system, viz., the inclination of the  $a$  to the  $c$  axis, or the value of the angle  $\beta$ .

The axial ratio and the value of  $\beta$  constitute the *crystallographic constants*. In orthoclase ( $\text{KAlSi}_3\text{O}_8$ ) these constants are  $\tilde{a} : \tilde{b} : c = .6585 : 1 : .5554$ .  $\beta = 63^\circ 56' 46''$ , and in augite (a calcium, magnesium, iron silicate)  $\tilde{a} : \tilde{b} : c = 1.09213 : 1 : .58931$ .  $\beta = 74^\circ 10' 9''$ .

In practice, although the choice of the lines that shall serve as the  $a$  and the  $c$  axes is purely arbitrary, it is usual to select those that will yield the simplest symbols for the forms most frequently found in combination. One or the other of these axes is made parallel to some prominent plane on the crystal or, better, to some prominent zone of planes. These planes then become pinacoids, domes, or prisms, when they are easy to recognize.

### HOLOHEDRAL DIVISION.

#### (*Prismatic Class.*)

**Pyramids.**—The most general symbol possible in the system is  $na : b : mc$ , or  $a : nb : c$ . Either represents a plane cutting the three axes at different distances, one of which, the intercept on  $b$  or on  $a$ , is considered as the unity on this axis. The presence of one of these planes necessitates the presence of a corresponding plane on the opposite side of the plane of symmetry, and the presence of these two demands the presence of two others parallel to the former because of the existence in the system of a center of symmetry. Consequently, *in this system the most general form consists of two pairs of planes*. If one of the planes occurs in an octant containing the acute angle  $\beta$ , all other planes of the

form occur also in acute octants. These are known as the *positive forms*. If, on the other hand, the planes are all in the obtuse octants, *negative forms* result.

A combination of a positive and a negative form whose planes possess the general symbol  $na : b : mc$ , gives a bipyramid (Fig. 185) analogous to the bipyramids in the orthorhombic system, except for the fact that it consists of two sets of planes of different shapes. Since *the positive and the negative forms each constitute half of this bipyramid, they are termed the positive and the negative hemipyramids*. A monoclinic bipyramid composed of the planes  $\bar{a} : \bar{b} : c$ , must thus be represented by two

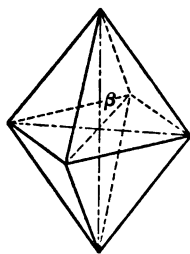


FIG. 185.—Combination of positive and negative hemipyramids.

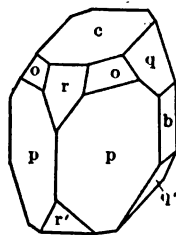


FIG. 186.—Crystal of ferrous sulphate.

symbols:  $+P$  and  $-P$ . Either of these hemipyramids may occur independently of the other, as each is completely holohedral. (See Figs. 186 and 188, p. 120.)

Of these hemipyramids there are three different kinds corresponding to the three kinds in the orthorhombic system. They are named in accordance with the same principles that determine the naming of the orthorhombic forms.

$\bar{a} : \bar{b} : c = +P(111)$  and  $-P(111)$ , unit hemipyramids.

$\bar{a} : n\bar{b} : c = +Pn$  and  $-Pn$ , orthohemipyramids.

$n\bar{a} : \bar{b} : c = +P\bar{n}$  and  $-P\bar{n}$ , clinohemipyramids.

When the parameter on the vertical axis becomes  $m$  there result other pyramids that belong to one of these three series, as  $\pm mP(hh\bar{l})$ ,  $\pm mPn(hk\bar{l})$ ,  $h > k$  and  $\pm mP\bar{n}(kh\bar{l})$ ,  $h < k$ .

When occurring alone the three hemipyramids can be dis-

tinguished from each other only by a comparison of their interfacial angles with the corresponding angles on the assumed ground-form. When in combination the *orthohemipyramids* and the *clino-hemipyramids* may often be distinguished from each other by the fact that the orthoforms occur near the terminations of the clinaxis, while the clinoforms occur near the terminations of the orthoaxis.

**Prisms.**—When the intercept  $m$  in the symbols  $mP$ ,  $mPn$ , and  $mP\bar{n}$  is made  $\infty$ , the pyramids which they represent become prisms, and we have the unit prism, the *orthoprism*, and the *clinoprism*.

Whether the prismatic faces are regarded as derived from the positive or from the negative hemipyramids, an inspection of figure 185 will show that *each of their planes must be at the same time in both a positive and a negative octant*. Hence the use of signs in the symbols of the prisms is not necessary.  $\infty P(110)$  is the unit prism,  $\infty P\bar{n}(hko, h > k)$ , the orthoprism and  $\infty P\bar{n}(kho, h < k)$ , the clinoprism. (See Figs. 188 and 189.)

**Domes.**—The domes in the monoclinic system, like those in the orthorhombic system, are composed of planes that are parallel to one of the lateral axes.

The *orthodomes* are parallel to the orthoaxis. There are two series: the positive orthodomes,  $P\infty(hol)$ ,  $mP\infty(hol)$ , etc.; and the negative orthodomes,  $-P\infty$ ,  $-mP\infty$ , etc., the former in the acute  $\beta$  and the latter in the obtuse  $\beta$  (see planes  $r$  and  $r'$  in Fig. 186).

The *clinodomes* are parallel to the clinaxis. These are at the same time in both positive and negative octants, and hence possess no signs. Their symbols are  $P\infty(ohl)$ ,  $mP\infty(ohl)$ , etc. (Plane  $q$  in Fig. 186).

**Pinacoids.**—The pinacoids are parallel to two of the axes at the same time. Their planes must belong to two octants, hence they possess no signs.

The *basal pinacoid* corresponds to the basal planes of other systems. Its symbol is the conventional  $oP(\infty 1)$ .

The other two pinacoids are the *orthopinacoid* and the *clinopinacoid*. The former is parallel to the vertical and the orthoaxis, and the latter to the vertical and the clinaxis. Their



symbols are, respectively,  $\infty P \infty (100)$  and  $\infty P \infty (010)$ . The clinopinacoid is the plane of symmetry (plane *b* in Fig. 186 and *M* in Figs. 187 and 188).

**Combinations.**—Combinations of monoclinic forms are more varied than are the combinations of the forms belonging to any system of a lower grade of symmetry. This is due to the fact that there are no closed forms in this system. The simplest crystals must be represented by at least two symbols.

In working out the symbols of monoclinic crystals it is absolutely necessary first to bring the crystals into the correct conventional position. The plane of symmetry must first be discovered, and then must be placed vertically and at the same time parallel

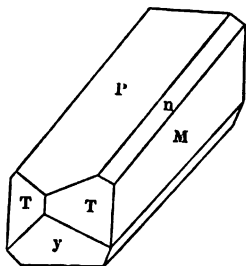


FIG. 187.

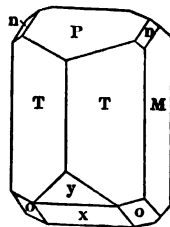


FIG. 188.

Crystals of orthoclase with  $\infty P \infty (M)$ ,  $0P(P)$ ,  $\infty P(T)$ ,  $2P \infty (n)$ ,  $+2P \infty (y)$ ,  $+P \infty (x)$  and  $+P(o)$ .

to the line of sight of the observer. When held in this position, the *b* axis runs horizontally from right to left. Its position is fixed by the symmetry of the system. The choice of the other two axes is a matter of convenience in each particular case and is regulated largely by the habit of the crystal. If this possesses planes that may be regarded as pinacoids, these planes serve to fix the positions of the two axes in question. The planes assumed as basal planes determine the position of the clinaxis.

The following illustration taken from Dr. Williams's Elements of Crystallography may serve to make these points clear. On the crystal of iron-vitriol ( $\text{FeSO}_4 + 7\text{H}_2\text{O}$ ), represented in figure 186, the only plane whose value is absolutely fixed is the plane of symmetry, or clinopinacoid (*b*). It is customary to make *c* the

basal pinacoid and  $p$  the fundamental prism, whence  $q$  becomes a clinodome,  $o$  a negative hemipyramid, and  $r'$  and  $r$  plus and minus hemiorthodomes. We might, however, turn the crystal so as to make  $c$  the orthopinacoid,  $q$  the prism, and  $p$  a clinodome; or we might even make  $r'$  the basal pinacoid, and  $r$  the orthopinacoid, when  $o$  would become the prism,  $c$  a hemiorthodome, and  $p$  and  $q$  both pyramids.

Figures 187 and 188 represent two common combinations in this system. The two crystals are very different in habit though

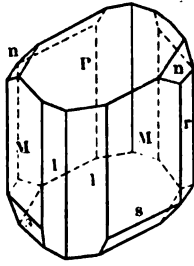


FIG. 189.—Crystal of real gas. See text for symbols of forms.

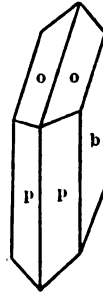


FIG. 190.—Crystal of gypsum, with  $\infty P$  ( $p$ ),  $\infty P \propto (b)$  and  $-P(o)$ .

they possess nearly the same forms. Both are crystals of the feldspar known as orthoclase ( $\text{KAlSi}_3\text{O}_8$ ). In the figures,  $M = \infty P \propto$ ,  $P = oP$ ,  $T = \infty P$ ,  $y = +2P \propto$ ,  $n = 2P \propto$ ,  $x = +P \propto$  and  $o = +P$ . Figure 189 is a crystal of realgar ( $\text{As}_2\text{S}_3$ ) with  $oP(P)$ ,  $\infty P \propto (r)$ ,  $\infty P(M)$ ,  $\infty P_2(l)$ ,  $P \propto (n)$  and  $+P(s)$ . Figure 190 is a crystal of gypsum ( $\text{CaSO}_4 + 2\text{H}_2\text{O}$ ) with  $\infty P(p)$ ,  $\infty P \propto (b)$  and  $-P(o)$ .

### HEMIHEDRAL DIVISION.

(*Domestic Class.*)

**Hemihedrism in the Monoclinic System.**—By the selection of half the planes of a hemipyramid in such a way that opposite ends of the ortho-axis are equivalently terminated, i.e., if each end is terminated by one plane, the conditions of hemihedrism for

this system are complied with. The resulting hemihedron still retains the plane of symmetry, but the axis of symmetry and the center of symmetry are gone (Fig. 191).

The new form derived from  $\pm P$  consists of the upper or the lower half of the hemipyramid, and is known as the *tetrapyramid*. Each hemipyramid, therefore, yields two hemihedrons, which are designated the upper and the lower tetrapyramids. Their symbols are  $+\frac{P}{2}u, +\frac{P}{2}l, -\frac{P}{2}u, -\frac{P}{2}l, \pm\frac{mP}{2}u.l, \pm\frac{mPn}{2}u.l, \pm\frac{mPn}{2}u.l$ , etc.

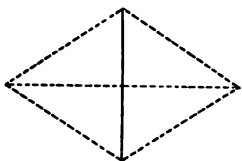


FIG. 191.—Diagram illustrating distribution of symmetry elements in monoclinic hemihedrons.

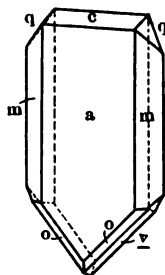


FIG. 192.—Crystal of  $K_2S_4O_6$  showing hemihedral forms. See text for symbols of forms.

In the same way the orthodomes yield upper and lower *tetraorthodomes*, the clinodomes yield upper and lower *hemiclino-domes*, and the prisms front and rear *hemiprisms*. The basal plane yields an upper and a lower form, and the orthopinacoid a front and a rear form.

Figure 192 is a combination of  $\frac{oP}{2}u(c), \frac{\infty P}{2}f(a), \frac{\infty P}{2}r$  ( $a$  behind),  $\frac{\infty P}{2}f(m)$  and  $\frac{\infty P}{2}r$  ( $m$  behind),  $\frac{P}{2}u(q), \frac{P}{2}l(v)$ , and  $+\frac{P}{2}l(o)$ , on crystals of potassium tetrathionate ( $K_2S_4O_6$ ).

**Hemimorphism** (*Sphenoidal Class*).—If one-half the planes of the hemipyramid are grouped at one end of the orthoaxis, the other half being absent, the plane of symmetry disappears, the

axis of symmetry becomes polar, and the center of symmetry no longer exists. The resulting form has but one element of symmetry which is the polar axis of binary symmetry (Fig. 193) and it is hemimorphic along the orthoaxis.

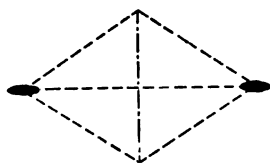


FIG. 193.—Diagram illustrating symmetry elements in monoclinic hemimorphs.

Each hemipyramid thus breaks up into two tetrapyramids, one of which is at the right end of the axis  $\left(+r\frac{P}{2}, -r\frac{P}{2}\right)$  and the other at its left end  $\left(+l\frac{P}{2}, -l\frac{P}{2}\right)$ .

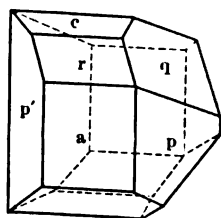


FIG. 194.

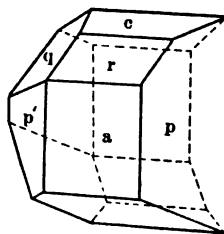


FIG. 195.

Right- (Fig. 194) and left- (Fig. 195) handed crystals of tartaric acid with  $\infty P \infty$  ( $a$ ),  $r\frac{\infty P}{2}$  ( $p$ ),  $l\frac{\infty P}{2}$  ( $p'$ ),  $\infty P$  ( $c$ ),  $-P \infty$  ( $r$ ),  $r\frac{P \infty}{2}$  ( $q$  in Fig. 194) and  $l\frac{P \infty}{2}$  ( $q$  in Fig. 195).

The prisms, the clino-domes, and the clino-pinacoids likewise may give rise to right and left forms, but the forms with planes that are parallel to the  $b$ -axis can yield no new hemimorphs.

*In all cases the corresponding right and left forms are enantiomorphous.*

On the crystals of tartaric acid ( $C_4H_6O_6$ ), represented in figures 194 and 195, the plane  $p$  is a plane of the prism at the right

end of the axis, or is  $r\frac{\infty P}{2}$ , and  $p'$  a similar plane at its left end or  $l\frac{\infty P}{2}$ . The plane  $q$  in figure 195 is  $l\frac{P\infty}{2}$  and  $q$  in figure 194  $r\frac{P\infty}{2}$ . The former is a left-handed crystal and the latter a right-handed crystal, so called because their solutions turn the plane of polarization of light to the left and the right, respectively.

## CHAPTER XI.

### THE TRICLINIC SYSTEM.

(*Pinacoidal Class.*)

**Symmetry of the Triclinic System.**—The forms belonging to the holohedral division of the triclinic system possess no planes of symmetry. There is, however, a center of symmetry. Each form, therefore, consists simply of a pair of parallel faces. Every pair of planes on triclinic crystals must therefore be represented by a different symbol.

**Axes.**—As there are no planes of symmetry in this system, the axes to which the forms are referred must be chosen arbitrarily. In the systems possessing three or more planes of symmetry *all* the axes are determined by the symmetry. In the monoclinic system there is only one plane of symmetry, hence the position of *one* axis only is determined. In the triclinic system there are no planes of symmetry. Consequently, *the position of no axis is fixed. Convenience alone determines the choice of axes.*

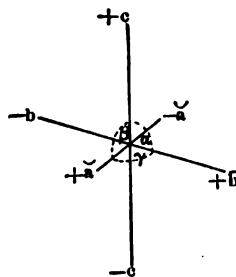


FIG. 196.—Axes of the triclinic system.

Practically, certain planes are assumed as prisms or pinacoids, and lines parallel to these are made the axes. Each substance crystallizing in this system has a different set of axes, which, however, when once established become the axes for all crystals of that substance.

The axes in the system differ, not merely in their unit lengths, but also in their inclinations. Each is inclined to the other two at the point of their common intersection. A model of the unit lengths of the triclinic axes would show three lines of different lengths intersecting each other at a common point, and obliquely inclined to one another (Fig. 196).

**Designation of the Axes.**—When the axes of a crystal are

once decided upon, one of them is held vertically as the *vertical axis*. Of the other two, the longer is made the *macroaxis*,  $b$ , and the shorter the *brachyaxis*,  $a$ . The macroaxis is made to incline downward toward the right and the brachyaxis downward toward the front. The signs used to designate the axis are the same as in the orthorhombic system. Thus the symbol of the axes in this system is  $\tilde{a} : \tilde{b} : c$ .

**Groundform and Crystallographic Constants.**—The groundform in this system consists of two parallel planes in opposite octants, each cutting the axes at the same distances, which, however, are different on the different axes. The intercepts of these planes are the unity lengths for the substance on which the form occurs. As in the other systems with unequal axes, the unities on  $a$  and  $c$  are measured in terms of the unity on  $b$ . This axial ratio varies for different substances.

In order that the position of the axes be fixed it is necessary to know their angles of inclination to one another, just as in order to describe the positions of the  $\tilde{a}$  and the  $c$  axes in the monoclinic system it is necessary to specify the value of the angle  $\beta$ . In the triclinic system each of the axes is inclined to the other two, hence it becomes necessary to determine the values of three angles. These are designated as the angles  $\alpha$ ,  $\beta$ , and  $\gamma$  (see Fig. 196). The angle  $\alpha$  is included between the vertical and the macroaxes. The angle  $\beta$  between the axes  $c$  and  $\tilde{a}$ , and the angle  $\gamma$  between the axes  $a$  and  $b$ .

The crystallographic constants in this system thus comprise six elements; viz.: the lengths of the three axes, and the angles  $\alpha$ ,  $\beta$ ,  $\gamma$ . For albite ( $\text{NaAlSi}_3\text{O}_8$ ) the crystallographic constants are  $\tilde{a} : b : c = .6338 : 1 : .5577$ , and  $\alpha = 94^\circ 3'$ ,  $\beta = 116^\circ 28 \frac{5}{6}'$ ,  $\gamma = 88^\circ 8 \frac{2}{3}'$ .

**Pyramids.**—The pyramids of the triclinic system differ but little in the nature of their planes from the pyramids in the monoclinic system. Their planes cut the three axes at different distances. If these distances are those that are assumed as the unity distances, the pyramid is a *unit pyramid*. If the intercept on one of the lateral axes is greater than unity on this axis when it is unity on the other, the pyramid is a *brachypyramid* or a *macro-*

*pyramid* according as the greater intercept is on the macroaxis or on the brachyaxis.

Since there is no plane of symmetry in the triclinic system, the existence on a crystal of one pyramidal plane necessitates the presence of no other corresponding plane except that which is parallel to it in the opposite octant, this being demanded by the center of symmetry. Consequently, a complete geometrical bipyramid comprises four pairs of parallel planes, each pair of which must be represented by a distinct symbol (Fig. 197). As each

pair of planes constitutes one-quarter of a complete pyramid, it is known as a *tetrapyrmaid*, and is represented in the unit series by  $P$  with an accent written near (Fig. 197) it in a position corresponding to the position of the front octant in which the plane occurs. Thus  $P'$  represents the form, one of whose planes is in the upper right-hand octant in front.  $P$ , the form whose front plane is in the lower right-hand octant,  $\bar{P}$  the form whose front plane is in the lower left-hand octant, and  $\bar{P}'$  the form one of whose planes is in the upper left-hand octant.  $\bar{P}'\bar{n}$  and  $\bar{P}'\bar{\bar{n}}$  are the corresponding *macro:tetrapyrmaids* and *brachytetrapyrmaids*. The indices of these forms are the same as those of the planes on the front of the pyramids of the tetragonal, orthorhombic, and monoclinic systems.

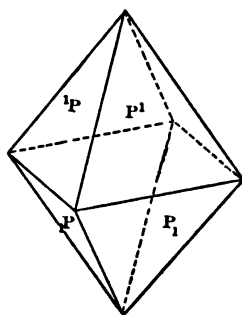


FIG. 197.—The four triclinic tetrapyrmaids.

*Prisms*.—Since the prisms are composed of planes parallel to the vertical axis, it necessarily follows that these planes are at the same time in two octants. Each prism consists of two parallel planes opposite each other. There are thus in this system *two hemiprisms* whose symbols are, respectively,  $\infty P'$ , and  $\infty \bar{P}'$  for the unit series,  $\infty \bar{P}'\bar{n}$  and  $\infty \bar{P}'\bar{\bar{n}}$  for the *macrohemiprisms* and  $\infty P'\bar{n}$  and  $\infty P'\bar{\bar{n}}$  for the *brachyhemiprisms*. The forms  $\infty P'$ ,  $\infty \bar{P}'\bar{n}$  and  $\infty P'\bar{\bar{n}}$  have planes on the right-hand side of the front of the crystal, and  $\infty \bar{P}'$ ,  $\infty \bar{P}'\bar{n}$  and  $\infty P'\bar{\bar{n}}$  have front left-hand planes. The form  $\infty P'$ , and the corresponding macro- and brachy-forms are derived from  $P'$  or from  $P$ , by the change of the intercept on  $c$  to  $\infty$ , and the



forms  $\infty'P$ , etc., from  $\cdot P$  or from  $\cdot P$  in the same manner. The corresponding indices of the hemiprisms are:

$$\infty P', 110; \infty P\tilde{n}, hko; \infty P\bar{n}, k\bar{h}o$$

$$\infty'P, 110; \infty'P\tilde{n}, h\bar{k}o; \infty'P\bar{n}, k\bar{h}o$$

**Domes.**—The macrodome planes that occur on the front of triclinic crystals lie either in the two upper octants or in the two lower ones. They may thus be regarded as derived from  $\cdot P$  and  $P'$  or from  $\cdot P$  and  $P\bar{n}$ . Each consists of a pair of faces, hence each corresponds to a half of an orthorhombic macrodome. Their symbols are  $\cdot P\infty$  for the upper front *hemimacrodome* and,  $\cdot P\bar{n}\infty$  for the lower front hemimacrodome (Fig. 198). Their corresponding indices are: *hol* and *h\bar{o}l*.

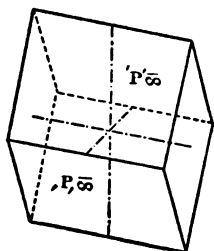


FIG. 198.—Two triclinic macrodomes.

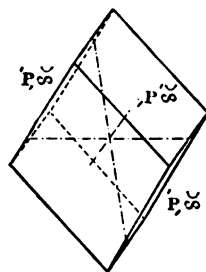


FIG. 199.—Two triclinic brachydomes.

Of the brachydomes there are also two, each consisting of a single pair of planes. Since their planes are parallel to the  $\tilde{a}$  axis, each is at the same time in two octants. The lower planes are parallel to the opposite upper ones. One of the planes is in the upper right-hand octant in front and in the corresponding octant behind. But the octant behind corresponds to the lower left-hand octant in front. Consequently, this dome face is regarded as derived from the pyramidal faces corresponding to  $\cdot P'$ , hence its symbol is  $\cdot P'\infty$  or *ohl*. This is the *upper right-hand hemibrachydome* or the *lower left-hand one*. Similarly  $\cdot P\infty$  is the upper left-hand brachydome or the lower right-hand one (Fig. 199), with the indices *o\bar{h}l*.

**Pinacoids.**—The pinacoidal planes are parallel to two axes, consequently each lies in four octants. There is only one brachy-

pinacoid, only one macropinacoid, and only one basal pinacoid possible on a triclinic crystal. Their symbols are, respectively,  $\infty P \sim$ ,  $\infty P \infty$ , and  $oP$ , or  $o10$ ,  $100$ , and  $oo1$ .

**Series of Pyramids and Domes.**—In the discussion above it is assumed that the pyramidal and dome faces cut the vertical axis at unity. In this system, however, as in the monoclinic and orthorhombic systems, there occur prisms and domes whose intercepts on the  $c$  axis are some multiple of unity. Their symbols may be represented by prefixing  $m$  to the symbols of the corresponding unit forms, or by indices that are the same as those which represent the front planes on corresponding forms in the orthorhombic system.

**Combinations of Forms.**—Every pair of faces in triclinic crystals demands a separate symbol for its representation, hence

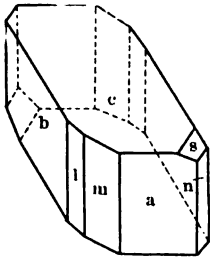


FIG. 200.—Crystal of copper sulphate. See text for symbols of planes.

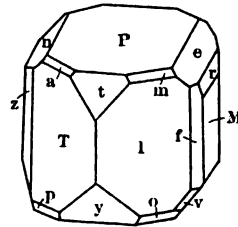


FIG. 201.—Crystal of anorthite. See text for symbols of planes.

the symbols of these crystals are usually long and somewhat complicated. The difficulty of deciphering triclinic crystals is increased by the fact that there is almost an unlimited choice in the selection of the axes, and that frequently the choice made for many crystals is not the most convenient. A few illustrations may indicate the principles made use of in fixing the axes. Figure 200 is a crystal of blue vitriol ( $\text{CuSO}_4 + 7\text{H}_2\text{O}$ ). Its planes are  $oP(c)$ ;  $\infty P \infty(a)$ ;  $\infty P \sim(b)$ ;  $\infty P'(n)$ ;  $\infty P'(m)$ ;  $\infty P_2(l)$  and  $P'(s)$ . Figure 201 represents a crystal of anorthite ( $\text{CaAl}_2\text{Si}_2\text{O}_8$ ). It is much more complicated than the blue vitriol crystal as it contains the forms  $oP(P)$ ;  $\infty P \infty(M)$ ;  $\infty P'(l)$ ;  $\infty P'(T)$ ;  $\infty P_3(f)$ ;  $\infty P_3(z)$ ;

$P'(m)$ ;  $P(a)$ ;  $P(o)$ ;  $P(p)$ ;  $4P, \bar{2}(v)$ ;  $P'\infty(t)$ ;  $2P, \infty(y)$ ;  $P'\infty(e)$ ;  $2P'\infty(r)$  and  $P, \infty(n)$ .

**Comparison of the Systems.**—The main differences between the crystallographic systems are conditioned by differences in the symmetry of the holohedrons belonging to them. The differences are expressed primarily in the character of the axes chosen for each system. For that system possessing the highest grade of symmetry the unities on all the axes are of equal length and the axes themselves are fixed in position, being perpendicular to one another. In the system possessing the next lower grade of symmetry there are four axes, one of which is of different length from the other three. All are fixed in position. The three axes of the same length are inclined to each other at angles of  $60^\circ$  and are perpendicular to the fourth axis. The system of the next lower grade has three axes, one of which is different in length from the other two which are equal. All are at right angles to one another. These three systems are all characterized by the possession of at least one principal plane of symmetry.

Of the systems having no principal plane of symmetry, that of the highest grade of symmetry possesses three axes of different lengths perpendicular to one another. The next system, which has only one symmetry plane, has only one of its axes fixed in position. This is perpendicular to the plane of symmetry. The other two axes are in the symmetry plane, but their positions in this plane are not fixed. Lines lying within this plane are arbitrarily chosen to serve as axes. The axes, therefore, are three unequal lines; one at right angles to the plane of the other two. The system with the lowest grade of symmetry has no axes fixed for it. They are chosen arbitrarily and must consist of three unequal lines inclined to one another at angles other than right angles.

The symbols for the axes become more and more complex as we pass from the systems of higher to lower grade, and more variable factors are concerned in them. These facts may be expressed by writing the symbols as follows, omitting angles of  $90^\circ$ :

<i>Systems.</i>	<i>Axes</i>
Isometric .....	$a$
Hexagonal .....	$a : c$
Tetragonal .....	$a : c$
Orthorhombic .....	$a : b : c$
Monoclinic .....	$a : b : c ; \beta$
Triclinic .....	$a : b : c ; \alpha, \beta, \gamma.$

## CHAPTER XII.

### CRYSTAL IMPERFECTIONS.

**Ideal Forms.**—Occasionally a crystal occurs which possesses the regularity of a well-made model. All the faces belonging to the same form have the same size. They are all equally developed and are all at equal distances from the center. These are ideal forms like those represented by most of the figures in this book. Usually, however, the regular and symmetrical growth of a crystal has been interfered with by some external agency or condition unfavorable to the symmetrical development of the ideal form. Moreover, it frequently happens that the faces of crystals are not perfectly plane as they are

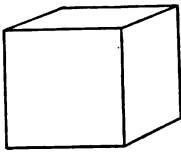


FIG. 202.



FIG. 203.

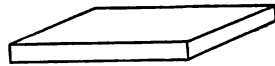


FIG. 204.

Symmetrical and distorted cubes with columnar (Fig. 203) and tabular (Fig. 204) habits.

assumed to be in crystallographic discussions, and occasionally the values of the interfacial angles vary from the calculated values for the ideal form. These are all imperfections in the crystals. They may be classified as imperfections in symmetrical development, imperfections in faces, and imperfections in angles.

**Distorted Crystals.**—The unequal rapidity of crystal growth in different directions often results in the elongation of the crystal in certain directions, and the consequent increase in the size of some of its planes at the expense of others. This process when carried to excess actually crowds out some of the faces that should

appear on the crystal, apparently destroys its symmetry, and produces bodies with the shapes characteristic of crystals of a lower grade of symmetry. The cube may grow much faster in the direction of one axis than in the direction of the others, and thus may simulate a tetragonal prism and basal plane, or it may grow rapidly in two directions, producing a tabular crystal (Figs. 202, 203, and 204). The dodecahedron of the isometric system

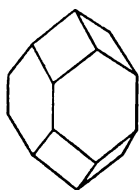


FIG. 205.

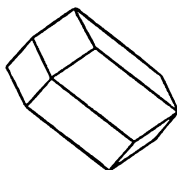


FIG. 206.



FIG. 207.

Dodecahedron distorted in such a way as to appear to possess tetragonal (Fig. 205), hexagonal (Fig. 206), and orthorhombic (Fig. 207) symmetry.

may by elongation in different directions come to resemble a tetragonal crystal (Fig. 205), an hexagonal one (Fig. 206), or an orthorhombic one (Fig. 207). By elongation in two directions, that is, by flattening, tabular crystals may be produced which at the first glance do not resemble in the slightest degree the forms from which they were derived. Figure 208, for instance, illustrates an octahedron that has been flattened in the direction of a line passing through the centers of opposite octahedral faces.

Nevertheless, however much crystals may be distorted in this way, their interfacial angles always have the values possessed by the corresponding ideal forms and their physical properties are the same. Moreover, by imagining the corresponding faces on all sides of the crystal to be moved toward its center, the form may be restored to the shape and symmetry properly belonging to it.

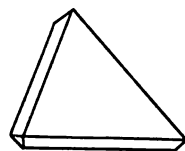


FIG. 208.—Flattened octahedron.

**Habit of Crystals.**—The habit of a crystal, or its general shape, depends largely upon the character of the distortion it has suffered. It is often so characteristic a feature of different substances that a number of terms have been invented for the purpose of describing it.

When a crystal grows much more rapidly in one direction than in others, and consequently is elongated in a single direction, it is said to have an *acicular habit* or to be *acicular*, when its cross section is very small; to be *prismatic* or *columnar*, when its cross section is greater. *Quartz* crystals are columnar in the direction of the *c* axis, *orthoclase* in the direction of the *a* axis, and *epidote* in the direction of the *b* axis. Figures 209, 210, and 211 represent crystals of these three substances.

When the elongation is in the direction of two crystallographic axes or, more properly, when the crystal is most largely developed laterally, it is *tabular*, as are many crystals of *mica*.

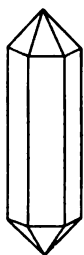


FIG. 209.

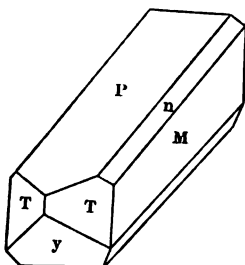


FIG. 210.

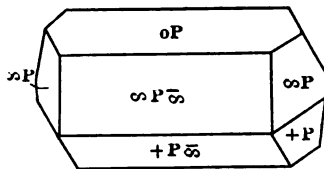


FIG. 211.

FIG. 209.—Columnar crystal of quartz elongated parallel to *c*.

FIG. 210.—Columnar crystal of feldspar elongated parallel to *a*.

FIG. 211.—Columnar crystal of epidote elongated parallel to *b*.

Other terms besides those defined are used in the descriptions of the habits of crystals, but they are self-explanatory and so need no special definition.

**Deformed Crystals.**—Distortions of another kind often produce very peculiar results, such as twisted or curved crystals, bent crystals, rounded crystals, etc., all of which are caused either by the prevalence of very unusual conditions during growth or by the action upon the crystal of mechanical forces after its formation. Many crystals imbedded in rocks are bent, and often they are cracked or shattered. Distortions often affect not only the shapes of the crystals, but often their interfacial angles as well. Figure 212 is a picture of a curved *hornblende*

crystal in a rock from near Marquette in Michigan. The rock is magnified 30 diameters.

**Imperfections in Crystal Planes.**—Although theoretically crystal faces are plane surfaces, practically a very large proportion



FIG. 212.—Curved crystal of hornblende in rock. (After G. H. Williams.)

of the crystal planes found in nature are not even. The unevenness may be due to striations, curvature, corrosion, or irregularity in growth.

**Striations.**—Striations are fine parallel lines that sometimes extend across all the faces of a crystal and at other times are

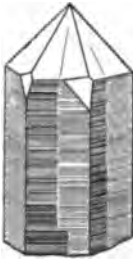


FIG. 213.—Striated quartz crystal.

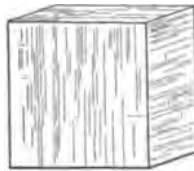


FIG. 214.—Striated crystal of pyrite.

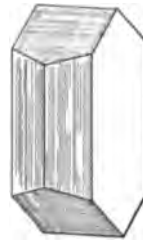


FIG. 215.—Striated crystal of plagioclase.

limited to the planes belonging to a single form. These may originate in the oscillatory combination of two distinct forms, as the horizontal striations on the prismatic faces of many *quartz* crystals (Fig. 213), which are produced by the oscillatory



combination of  $\infty R$  and  $R$ , or on cubes of *pyrite* (Fig. 214) due to combinations of  $\infty O \infty$  and  $\left[ \frac{mO \infty}{2} \right]$ . They may also be due to polysynthetic twinning, as in the case of striations on the prismatic, the basal, and the macrodome faces of *plagioclase* crystals (Fig. 215), when the twinning lamellæ are so small that they can with difficulty be recognized. (See p. 155-156.)

**Curvature.**—Many substances, like *diamond* (C) (Fig. 216), *dolomite* ((CaMg)CO<sub>3</sub>), etc., are so frequently bounded by curved faces that their occurrence must be ascribed to some characteristic property of the molecules of which these bodies are composed.

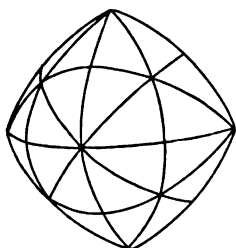


FIG. 216.—Crystal of diamond with curved faces.



FIG. 217.—Natural etched figures on diamond crystal. (After Tschermak.)

**Corrosion and Etched Figures.**—After a crystal has been formed with perfect planes, solutions may attack it and dissolve portions of the faces, rounding their edges and so causing the plane to become curved, or else pitting them with little hollows, known as etched figures. The etched figures vary in shape in different minerals and on different planes of the same mineral, their shapes always being governed by the symmetry of the plane on which they occur. Those on the cubic faces of *diamond* (Fig. 217) are hopper-shaped.

**Imperfections Caused by Irregularities in Growth.**—The irregularities developed during the growth of a crystal are too numerous to be specified. Often an obstruction met with by the crystal will become imbedded in it and so cause one or more of its planes to become distorted, or it may impress its shape upon

a plane with which it comes in contact and in this way destroy its perfection of surface. Or a crystal may grow rapidly, forming its edges first and building up a skeleton, which may not become entirely filled up. In place of a crystal face there may thus result a reversed pyramid, as in the so-called hopper-shaped crystals of *salt*, crystals of *cuprite* ( $\text{CuO}$ ) (Fig. 218), or *galena* ( $\text{PbS}$ ), and in metallic bismuth (Fig. 219).

**Symmetry of Imperfections.**—Although the imperfections in crystal faces tend to destroy their ideal development, these imperfections are nevertheless governed in a great measure by the symmetry of the crystal form. *All crystallographically equivalent*



FIG. 218.—Skeleton crystal of cuprite.



FIG. 219.—Portion of skeleton crystal (cube) of bismuth.

*planes are similarly affected.* The symmetry of the imperfections on forms in combinations or even in apparently simple forms may often serve to aid in the correct determination of their symmetry. In this way certain forms that appear to be holohedrons are shown to be in reality hemihedrons that are not geometrically distinguishable from the holohedrons from which they are derived.

The striations on cubes of *pyrite* ( $\text{FeS}_2$ ) (see Fig. 214) are often arranged symmetrically with respect to the three principal planes of symmetry passing through the cube, and with respect to these only. The cube is thus symmetrical in the same way as are the hemihedrons derived by parallel hemihedrism, hence the form may be regarded as a hemihedral cube derived from the holo-

hedral cube by the parallel method of hemihedrism. When in combination with distinctive hemihedrons, these are always either the diploid or the pyritoid, the characteristic hemihedrons of parallel hemihedrism, and never forms derived by the gyroidal or the inclined methods of hemihedrism.

Again, on the mineral *sphalerite* (ZnS) there frequently occur eight triangular faces distributed like the eight faces of an octahedron (Fig. 220). But of these, four are smooth except for a few



FIG. 220.—Crystal of galena, showing distribution of imperfections.

striations, and four are rough, a smooth and a rough face alternating. The rough faces correspond to the faces of a tetrahedron and the smooth ones to those of a corresponding tetrahedron with an opposite sign. If the former be considered  $+\frac{O}{2}$ , the latter is  $-\frac{O}{2}$ .

These forms are inclined hemihedrons, hence we should expect all other forms on this same mineral to be inclined hemihedrons with characteristic forms, or holohedrons that do not yield new forms by the inclined method of hemihedrism. As a matter of fact, these are the only forms that do occur in the mineral.

**Variations in Crystal Angles.**—The imperfections in the planes on crystals do not often affect the values of their interfacial angles. When these vary the variations are usually due to a difference in the chemical composition of the minerals whose angles are being compared, to differences in the temperature at which the crystals are measured, or to some mechanical force acting upon the crystal from without. The variations are always slight and, except in the case where mechanical force is concerned, the variations only *emphasize the fact that the interfacial angles on crystals are their most characteristic features.*

**Impurities within Crystals.**—Ideal crystals consist of homogeneous matter possessing the same chemical composition throughout and having the color of the pure substance that composes their mass. Departures from this ideal purity are frequently met with.

The impurities may consist of a dilute coloring matter, or pigment, which is present in such small quantity as to be indeterminate, but nevertheless in sufficient quantity to give a decided color to the crystals. Pure *quartz* ( $\text{SiO}_2$ ) is colorless, smoky quartz is gray or black, but the coloring matter in it is present in such small quantity that its true nature is not certainly known. Sometimes the coloring matter is evenly distributed throughout the entire crystal, sometimes it is regularly distributed in concentric layers, called zones, and sometimes it is irregularly distributed without respect to crystallographic directions. *Tourmaline* crystals from Hebron and Mount Mica, Maine, very often have a pink nucleus surrounded by zones of different shades of green. Figure 221 represents a cross section through such a crystal.



FIG. 221.—Cross section of crystal of tourmaline exhibiting zonal arrangement of color.

The other impurities in crystals may be classed together as inclusions. They can usually be distinguished from the mineral substance in which they are imbedded, if not by the eye, at least under the microscope. They comprise gas, fluid, glass, crystal, and unindividualized inclusions.

*Gas inclusions* are found in crystals of both aqueous and igneous origin. They appear as little cavities of different shapes, in some cases scattered promiscuously through the crystal substance; in others arranged zonally. When the cavities have the shapes of the crystal they are known as *negative crystals*. Their contents may be air, carbon dioxide ( $\text{CO}_2$ ), marsh gas ( $\text{CH}_4$ ), or some other hydrocarbon, sulphur dioxide ( $\text{SO}_2$ ), or some simple gas.

*Fluid Inclusions*.—When the pores are filled with liquid they are known as fluid cavities. Like the gas inclusions these may be of different shapes, and, like them, they may be arranged regularly or irregularly within the crystal substance. The fluid enclosed in the cavities may be water, liquid carbon dioxide, some other liquid gas, petroleum, or some dilute salt solution. Often the liquid encloses a bubble of gas (Fig. 222) which by its movements

serves to distinguish the fluid from the glass inclusion. Figure 223 is a magnified view of gas and liquid inclusions in quartz.

In a few cases two fluids which do not mix are enclosed in the same cavity in which case that one whose surface tension is greatest occupies the center of the cavity, while the other surrounds it as an envelope (Fig. 224). When liquid and gas are both enclosed in the same cavity, the size of the gas bubble may be made to change by increasing or diminishing the temperature, and at the proper temperature may be made to disappear entirely. By noting the temperature at which the bubble disappears, i.e., the

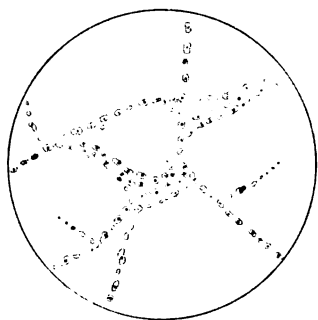


FIG. 222.—Liquid inclusions in quartz. Slightly magnified.

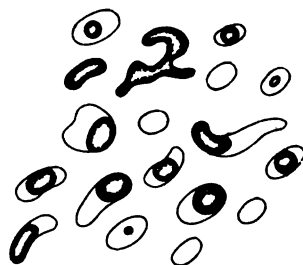


FIG. 223.—Liquid inclusions in quartz. Greatly magnified.

temperature at which there is no distinction between the liquid and the gas, the nature of the inclusion may sometimes be determined.

These phenomena are supposed to show that the substance was crystallized under high pressure, and that while growing it surrounded a portion of the mother liquor from which it was separating.

*Glass inclusions* occur in crystals that have solidified from a molten mass and have enclosed a portion of this mass during growth, or in those which have since their formation been heated so high as to melt foreign substances contained within them. These inclusions also often contain bubbles, but in this case they are immovable. The existence of two bubbles in the same inclusion is proof that the cavity is filled with glass.

*Crystal inclusions.*—These are minute crystals caught up and enclosed in larger crystals during growth, or minute crystals formed by the decomposition of the latter.



FIG. 224.—Thin section of smoky quartz showing inclusions consisting of central gas bubble, surrounded by two liquids that do not mix. The outer liquid is probably water and inner liquid and gas bubble carbon dioxide. Magnified about 150 diam. (After Rosenbusch.)

When a crystal of *orthoclase* solidifies from a solution in which there occur already formed tiny crystals of *apatite*, some of these small crystals may become embedded in the orthoclase as inclusions. On the other hand, when orthoclase under the influence

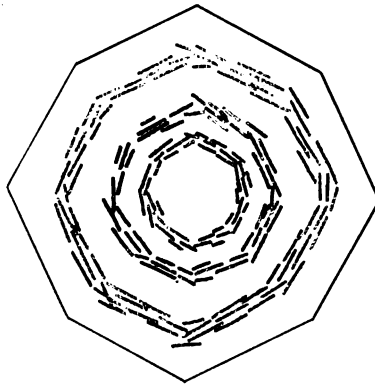


FIG. 225.—Section of leucite crystal showing zonal arrangement of minute inclusions. (After Tschermak.)

of chemical agents begins to change into *kaolin*, the first stages of the alteration are seen in the presence of tiny scales of kaolin scattered through the mass of the orthoclase. In some crystals the inclusions are so arranged as to produce an iridescence in the

enclosing substance, as in the case of *labradorite*, *bronzite*, etc. In others they are arranged zonally, as in *leucite* (Fig. 225). When so small as to baffle attempts to identify them as definite mineral species, the inclusions are known as *crystallites*; when large enough to be identified, they are referred to as *microlites*.

The inclusions in crystals can best be studied in very thin slices. When these are viewed under the microscope it is found that very few crystals are free from impurities of many kinds. Nearly all contain either liquid, gas, or mineral inclusions. The former are especially abundant even in crystals occurring in the hardest rocks. Often in a single grain (Fig. 222) of the quartz in a granite will be seen hundreds of tiny pores nearly filled with liquid. In many of these are little bubbles that move slowly through the liquid mass or dance rapidly, as the case may be, moving incessantly, in consequence, probably, of the slight changes of temperature to which the substance containing them is subjected. Sometimes, but not as commonly, a liquid inclusion will contain a little crystal of some salt, which may be made to dissolve by warming the specimen or to grow larger by cooling it.

The quantity of liquid enclosed in some minerals is very large, reaching, it is said, in the case of the constituents of certain rocks as much as 1.8 per cent. of their volume. Often the liquid is really a condensed gas, most commonly liquid carbon dioxide, methane, or nitrogen. The volume of gases obtained from enclosures in quartz crystals from Poretta, Italy, amounted in one instance to .03 per cent. of the volume of the quartz. The total quantity of gas that has been obtained from certain other crystals is much greater, reaching 13 per cent. by volume in some cases, but much of this was present in some other form than as enclosures in cavities.

## CHAPTER XIII.

### CRYSTAL AGGREGATES.

**Crystal Individuals and Crystal Aggregates.**—Thus far only *crystal individuals* have been discussed, since these most nearly exhibit the ideal forms demanded by symmetry. Rarely, however, are crystal individuals complete. There is usually lacking some one or more of their faces where the crystal was attached during its growth.

Individual crystals that exhibit only an occasional face and crystallized masses that possess forms due to influences acting from without the crystallizing body (allotriomorphic forms) are distinguished as *crystal grains*.

Neither crystal individuals nor crystal grains commonly occur isolated. They are usually grouped together irregularly. Often, however, several crystals or parts of crystals are grouped in parallel positions, or in such a way that the different individuals are separated from one another by a plane about which they are symmetrically disposed. The former of these definite groupings are known as *parallel growths* and the latter as *twinned crystals*. The general term "*aggregate*" is given to the irregular groupings.

**Irregular Aggregates.**—Irregular aggregates may consist of grains of the same mineral irregularly grouped, as are the particles of calcite in a coarse-grained limestone (Fig. 226), or they may be composed of grains of several substances, as is the aggregate of orthoclase ( $\text{KAlSi}_3\text{O}_8$ ), quartz ( $\text{SiO}_2$ ), and mica known as granite (Fig. 227). Such aggregates as these, where each grain is completely or almost completely bounded by other grains, are distinguished as *crystalline aggregates*.

When the grains in an irregular aggregate are partially bounded by their own planes, the aggregate is a *crystal aggregate*; and when the individual grains approach in form the character of crystal individuals, the aggregate is a *crystal group*.



Figure 227 is from an enlarged photograph of a thin slice of granite. It shows the minerals crowded together irregularly.

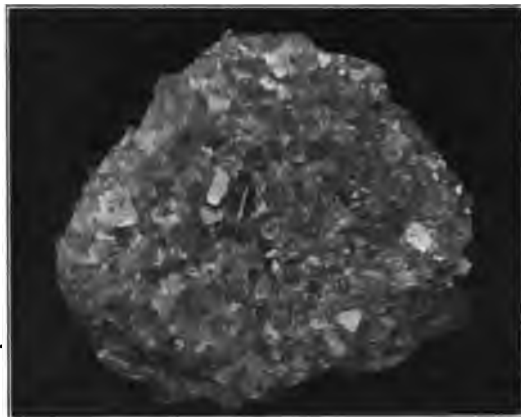


FIG. 226.—Piece of white marble, a crystalline aggregate of a single mineral (calcite). The bright areas are due to reflections from the cleavage surfaces of individual grains.

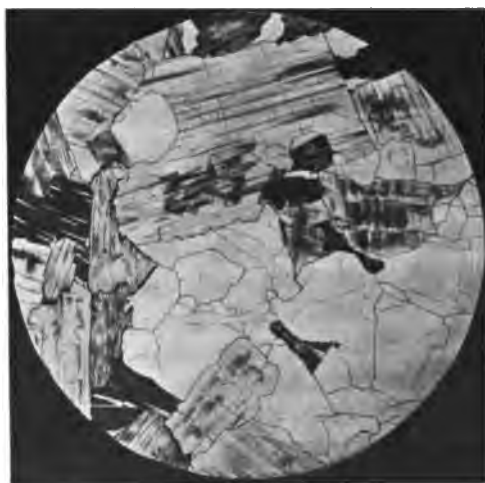


FIG. 227.—Thin section of granite, illustrating a granular aggregate of several different minerals, viz: quartz (clear), soda-feldspar (striped), potash feldspar (cross-barred) and biotite (black). Magnified about 5 diameters.

None of them possess crystal forms. Figure 1 (page 3) is a crystal group, composed of well-defined crystal individuals

of calcite, each bounded by its own planes except at its point of attachment.

**Classification of Crystalline Aggregates.**—The classification of crystalline aggregates is not in itself a matter of great importance. In discussing aggregates of crystal particles, however, it is convenient to make use of terms that will indicate briefly what would in ordinary language require a clause for its description. For descriptive purposes, then, we may classify aggregates with respect to the size of their constituent grains, with respect to their manner of development, and with respect to the strength of their cohesion.

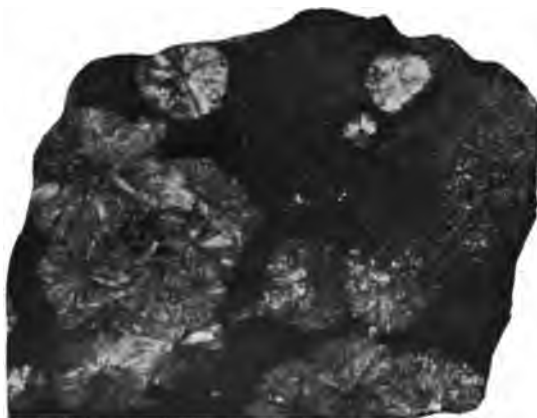


FIG. 228.—Radiating groups of wavelite crystals on a rock surface.

According to the size of the component grains, an aggregate may be *phanero-crystalline* when its particles are large enough to be seen by the naked eye, or *crypto-crystalline* when they may be detected only with the aid of a microscope. Phanero-crystalline aggregates may be coarse-grained, medium-grained, or fine-grained.

According to the manner of development of its individual components, an aggregate may have a structure, which is described as *granular*, when the particles are about equally developed in all directions, as are the grains in a granite or a coarse limestone; *lamellar*, or platy, when the grains are scaly, platy, or tabular,

as in the case of a mica-schist; or *fibrous*, when the components are all acicular, as in the case of *asbestos*  $((\text{MgCa})\text{SiO}_3)$ .

The components of a fibrous aggregate often tend to group



FIG. 229.—Globular mass of limonite fibers.

themselves around certain centers from which they radiate, producing masses that imitate the shapes of common objects. When the fibers radiate from a point and are confined to a single



FIG. 230.—Botryoidal groups of limonite fibers.

plane, like the spokes in a wagon wheel, they produce *radial aggregates* (Fig. 228). When the radiation is outward in all directions from a point, *globular forms* result (Fig. 229). These

latter in turn may aggregate, forming a bunch of globules, when its form is known as *botryoidal* (Fig. 230). Often the radial grouping is incomplete or the fibers may not diverge regularly. In this case a *sheaf-like* bundle may result, as in the case of the



FIG. 231.—Sheaf-like group of stilbite crystals.

mineral *stilbite* (calcium aluminium silicate with water) (Fig. 231).

When the fibers radiate from a line they produce a *stalactitic* growth, or a *stalactite* (Figs. 232 and 233).



FIG. 232.—Stalactite of limonite.

If the cohesion between the particles of an aggregate is strong, whatever may be the shape of the aggregate, it is said to be *compact*; when the cohesion is slight, the aggregate is *friable*.

**Parallel Growths.**—Frequently crystals are so grouped that

one or all of the axes of the different individuals are parallel. The most complete parallelism occurs in the case of individuals of the same substance. Often two crystals of the same substance occur side by side in contact, with all the axes of the one parallel



FIG. 233.—Cross section of group of stalactites of limonite and goethite (hydroxides of iron.)

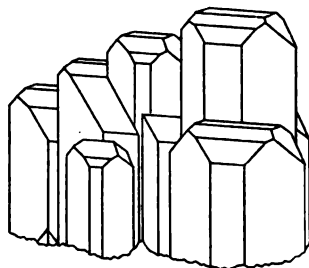


FIG. 234.—Group of barite crystals with axes parallel. (After *Tschermak*.)

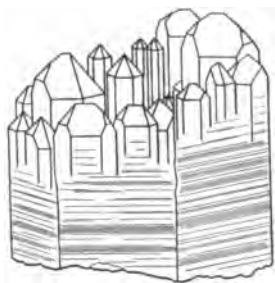


FIG. 235.—Group of quartz crystals merging into single crystal below. (After *Tschermak*.)



FIG. 236.—A number of small crystals of quartz grouped to form a single large crystal.

to the corresponding axes of the other, as in the case of *barite* ( $\text{BaSO}_4$ ) (Fig. 234). Sometimes the contact surfaces are small, when the crystals just touch. At other times the crystals appear to penetrate one another, only a portion of each crystal being

visible. Figure 235 is a group of *quartz* crystals the lower portions of which are merged into a single crystal. In some cases many small individuals (sub-individuals) are grouped side by side, forming a large crystal (Fig. 236), or are arranged along a common axis in such a way that only the individual at the two ends of the group exhibit complete faces (zircon,  $\text{ZrSiO}_4$ , Fig. 237), while the intermediate individuals appear as thin plates or lamellæ crowded between these. When the lamellæ of the intermediate individual are very thin they produce the effect of striations on the faces of a single individual. These striations are caused by the repetition of the interfacial edges on consecutive individuals—a condition known as an oscillatory combination.

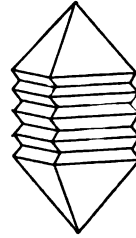


FIG. 237.—Zircon crystal composed of several individuals arranged in a vertical pile.

Again many minerals form branching aggregates, the branches of which are composed of many small crystals arranged in parallel position (Fig. 238). The angles between the branches correspond in value to the angles between the crystallographic axes of the little individuals or to some other equally characteristic angles. These are often known as

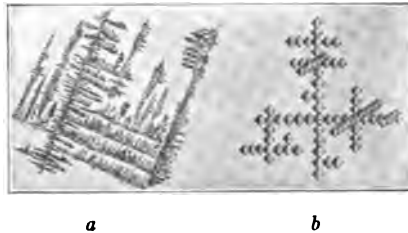


FIG. 238.—Skeleton crystals of argenteite in section parallel to cubic face (a). Portion of same magnified (b). Showing grouping of tiny octahedrons arranged in lines parallel to the crystallographic axes. (After Tschermak.)

*dendritic growths*. They occur in *moss agate* and in many glassy volcanic rocks.

A *druse* is a crust composed of small crystals implanted side by side in approximately parallel positions upon some other mineral or upon a rock surface (Fig. 239).

More frequent even than the parallel growth of similar mineral species is the case of parallel growths of crystals of different substances, which are, however, analogous in composition. Substances like *calcite* ( $\text{CaCO}_3$ ) and *magnesite* ( $\text{MgCO}_3$ ) possess similar though not identical crystal forms. Such substances are known as isomorphous substances (see page 229). These often crystallize together so that the axes of the crystals of both substances are approximately parallel. Often the crystallization is so intimate that the different substances cannot be detected even under high powers of the microscope. Calcite, for instance, often



FIG. 239 —Coating, or druse, of small crystals of smithsonite ( $\text{ZnCO}_3$ ) on massive smithsonite.

contains small quantities of magnesium, which may occur in the compound as intermixed magnesite. At other times a nucleus of one substance may be surrounded by an envelope of another substance, and this in turn by an envelope of still a third substance. A cross section through such a growth will show a nucleus surrounded by concentric zones of varying composition. This is known as a *zonal growth*. It is well exhibited in the *garnets*, the *feldspars* (Fig. 240) and the *pyroxenes*, where the difference in composition of the different zones is indicated by a difference in color or by the effect of the different layers upon polarized light.

A third class of partial parallelism is noted in the association of mineral species of entirely different compositions, possessing

entirely different crystal forms. Little prismatic crystals of *rutile* ( $\text{TiO}_2$ ), for instance, are often implanted on tabular crystals of *hematite* ( $\text{Fe}_2\text{O}_3$ ) in such a way that the vertical axes of the rutile are parallel to one of the lateral axes of hematite (Fig. 241).

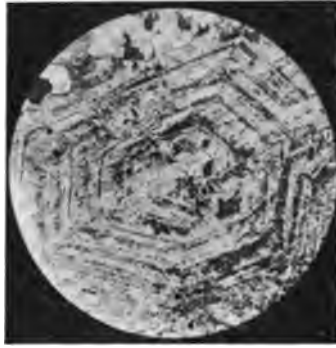


FIG. 240.—Section of zonal orthoclase in rock. Magnified  $7\frac{1}{2}$  diameters. (After Rosenbusch.)

Many other instances of a like association of different mineral species are known, as, for instance, pyrite (isometric,  $\text{FeS}_2$ ) on *marcasite* (orthorhombic  $\text{FeS}_2$ ) (Fig. 242) and *albite* ( $\text{NaAlSi}_3\text{O}_8$ ) on *orthoclase* ( $\text{KAlSi}_3\text{O}_8$ ) (Fig. 243).



FIG. 241.—Small crystals of rutile attached to crystal of hematite.

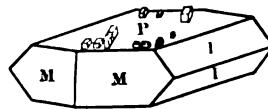


FIG. 242.—Little crystals of pyrite implanted on marcasite. (After Linck.)

**Twinned Crystals.**—Often two or more crystals or parts of crystals are so grouped that they are symmetrical to each other with respect to a plane between them, that is not a plane of symmetry for either individual. These groups, or twinned crystals (Fig. 244), are of great importance crystallographically,



as the nature of the twinning is very characteristic for different substances.

**Twinning Plane.**—The twinning plane is the plane about which the twinned individuals are symmetrical (plane ABCD in Fig. 244). It can never be a plane of symmetry for the individuals, for in this case a parallel growth would result, but it may be any other crystallographic plane possible. Usually the twinning plane is one with very simple indices.

The *twinning axis* is a line about which one of the twinned individuals or parts of individuals may be supposed to be revolved and brought into a parallel position with the other. The twinning axis is usually normal to the twinning plane. The plane of

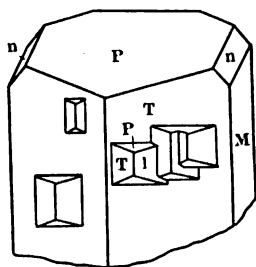


FIG. 243.—Crystals of albite on orthoclase with vertical axes of both parallel. (After Linck.)

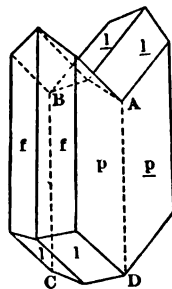


FIG. 244.—Contact twin with twinning plane and composition faces the same, viz.: ABCD.

union between the two twinned parts is the *composition face*, and this may or may not be coincident with the twinning plane. In most cases of twinned crystals there is observed a re-entrant angle between certain of the contiguous planes on opposite sides of the composition face, which angle serves as a distinguishing mark of twinning (see top of crystal in Fig. 244).

**Contact Twins.**—When parts of two crystal individuals are united in a plane in such a way that practically all of each individual is on one side only of the plane, the twin is known as a contact twin.

In figure 244, which represents a contact twin of monoclinic

*gypsum* ( $\text{CaSO}_4 + 2\text{H}_2\text{O}$ ), the twinning plane is parallel to  $\infty P \infty$ , which is also the composition plane. In *albite*, which is triclinic (Fig. 245), the twinning plane is  $\infty P \infty (M)$ , and the composition plane the same. In *orthoclase*, which is monoclinic (Fig. 246), the twinning plane is  $\infty P \infty$ , and the composition face is  $\infty P \infty (M)$  (corresponding to  $\infty P \infty$  in albite).

**Penetration Twins.**—Frequently the twinning individuals penetrate each other, so that they cannot be said to have any composition face—they are apparently grown through one another. Such twins are known as penetration twins.

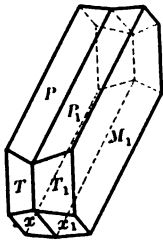


FIG. 245.

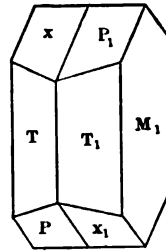


FIG. 246.

Twins of feldspars possessing different twinning planes but corresponding composition faces. The twinning plane in Fig. 245 coincides with the composition face. In Fig. 246 the two are at right angles.

Figure 247 illustrates an interpenetration twin of *fluorite* ( $\text{CaF}_2$ ) in which two cubes are so placed that they are symmetrical about a plane parallel to an octahedral face. Figure 248 is an interpenetration twin of *orthoclase* with the same law of twinning as in figure 246; i.e.,  $\infty P \infty$  is the twinning plane and  $\infty P \infty$  the composition face.

When the parts of such twins are of a low grade of symmetry and by their intergrowth tend to produce a body with a higher grade of symmetry, the resulting twin is known as a *supplementary twin*. Thus positive and negative hemihedrons by twinning may produce a form with the geometrical symmetry of a holohedron. Figure 249 represents the twinning of two right-handed tetartohedral crystals of *quartz* reproducing a form with hemihedral symmetry.

**Repeated Twins.**—Although the term twin suggests that twinned crystals consist of two parts in the twinning relation with respect to one another, it nevertheless often happens that to the

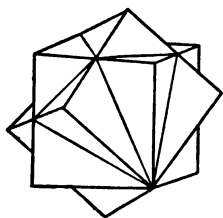


FIG. 247.—Interpenetration twin consisting of two cubes of fluorite twinned about an octahedral plane.

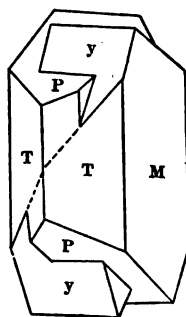


FIG. 248.—Interpenetration twin of feldspar with same twinning plane and composition face as in Fig. 246.

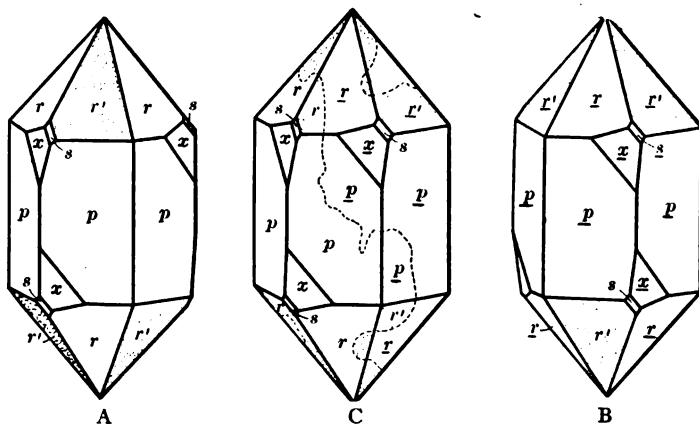


FIG. 249.—Supplementary twin of quartz (C), produced by combination of two right-hand crystals (A and B) of which one with respect to the other is revolved about the  $c$  axis through an angle of  $60^\circ$ . (After Groth).

two a third, a fourth, etc., are added, each possessing a twinned relation to those contiguous to it. Thus *trillings*, *foublings*, etc., are produced.

Repeated twinning may take place in either one of two ways.

The twinning planes between the contiguous individuals may be parallel to each other or they may not.

In the former case the alternate individuals are in parallel position, as is indicated in figure 250, which represents a series of five orthorhombic *aragonite* ( $\text{CaCO}_3$ ) plates, twinned about parallel planes of  $\infty P$ . A cross section of such a twin is pictured in figure 251.

Twins of this kind are known as *polysynthetic* twins. Each plate is called a *lamella*, and successive lamellæ are in reversed positions with respect to their neighbors. Fig. 252 is a polysynthetic twin of albite twinned in the same way as is represented in Fig. 245 (page 153).

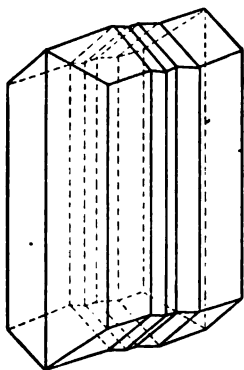


FIG. 250.

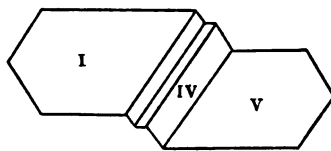


FIG. 251.

Repeated twin of aragonite (Fig. 250) and cross section of same (Fig. 251) showing successive twinning planes parallel.

Often a series of very thin twinned lamellæ (Fig. 253) produces a striation on the faces of crystals or of cleavage pieces. This striation is well exhibited by cleavage pieces of triclinic feldspar (Fig. 254). Light will be reflected in the same direction by alternate lamellæ, so that, when the specimen is held in such a way as to catch the light from a distinct source, one set of lamellæ will appear glistening while the intermediate lamellæ are dull. Upon turning the specimen through a slight angle the reflecting lamellæ become dull while those that were originally dull become bright. This phenomenon is the oscillatory twinning referred to on page 136.

When the twinning planes between the successive individuals of a repeated twin are alternate planes belonging to the same form, the twin turns on itself, producing a circular or wheel-shaped group, known often as a *cyclic* twin.

Figure 255 is an illustration of an *aragonite* trilling in which the two symmetrical planes of  $\infty P$  become successively the twinning plane. By further repetition of this twinning a nearly

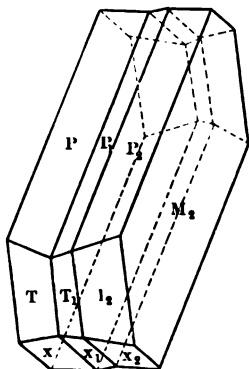


FIG. 252.—Polysynthetic twin of albite with thin lamella in center.

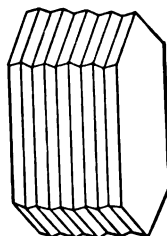


FIG. 253.—Diagrammatic sketch of polysynthetic twin of albite with many lamellæ.

complete circle or hollow cylinder may be formed. Figure 256 is the cross section of such a repeated twin in which four individuals are twinned.

The examples of repeated twinning given above are all



FIG. 254.—Twinning striations on cleavage surface of oligoclase. The bands are due to the alternation of lamellæ that reflect light differently. Natural size.

illustrations of contact twins. Interpenetration twins are also repeatedly twinned, the groups taking the form of a rayed star or of a bundle of symmetrical plates.

Figure 257 is a repeated twin of *cerussite* ( $\text{PbCO}_3$ ) an ortho-

rhombic mineral, in which the brachy-prism  $\propto P\bar{3}$  is the twinning plane; and figure 258, a twin of the orthorhombic *chrysoberyl* ( $\text{BeAl}_2\text{O}_4$ ), with the brachydome  $P\infty$  the twinning plane. Figure 259 represents a very complicated interpenetration twin

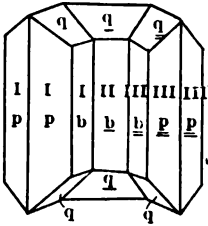


FIG. 255.—Cyclic trilling of aragonite with successive twinning planes alternate  $\propto P$  faces. Compare Fig. 250.

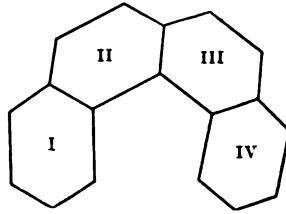


FIG. 256.—Cross section of aragonite fourling with successive twinning planes alternate  $\propto P$  faces. Compare Fig. 251.

of monoclinic crystals of *phillipsite* ( $(\text{CaK}_2)\text{Al}_2\text{Si}_5\text{O}_{14} + 5\text{H}_2\text{O}$ ). The three columnar portions of the group are twinned about the same plane (A-A). Each column is composed of what are apparently two individuals twinned about another plane (B-B),

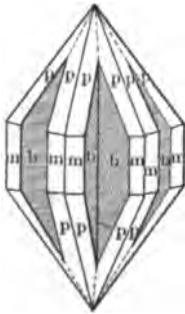


FIG. 257.—Repeated interpenetration twin of cerussite.

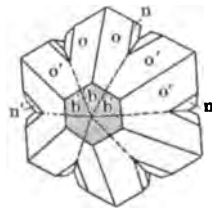


FIG. 258.—Repeated interpenetration twin of chrysoberyl.

but each of these seeming individuals is in reality a combination of two individuals twinned about a third plane. In the group there are, therefore, twelve individuals twinned according to three different laws.

**Mimicry.**—By the repeated twinning of crystals of a low grade of symmetry a group is often produced which appears to be a simple crystal of a higher grade of symmetry than that of its

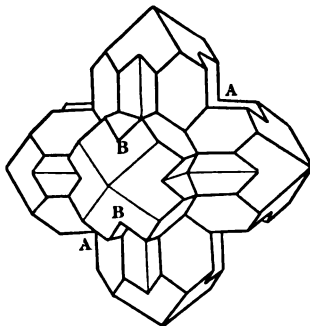


FIG. 259.—Complicated interpenetration twin of phillipsite. Twelve individuals twinned. See text for explanation.

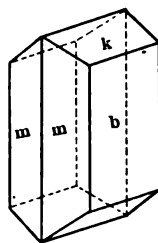


FIG. 260.—Simple crystal of aragonite (orthorhombic) with  $\infty P(m)$ ,  $\infty P(b)$  and  $P(k)$ .

components. Thus the orthorhombic mineral *aragonite* ( $\text{CaCO}_3$ ) often crystallizes in prismatic crystals, bounded by the forms  $\infty P(m)$ ,  $\infty P(b)$ , and  $P(k)$  (Fig. 260). A cross section

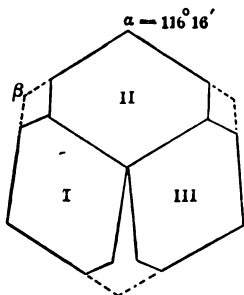


FIG. 261.—Cross section of trilling of aragonite with twinning plane alternate  $\infty P$  faces. See also Fig. 256.



FIG. 262.—Trilling of aragonite producing a group that resembles an hexagonal prism terminated by basal planes. A reentrant angle is seen on the right. Natural size.

through a single crystal has the outline shown in one of the individuals in figure 261, in which the angle between the  $\infty P$  faces is  $116^\circ 16'$ . Trillings formed of three of these crystals twinned

parallel to  $\infty P$  have a cross section like figure 261. When the spaces between the dotted lines in the figure and the body of the crystal become filled with mineral substance, the trilling strongly resembles an hexagonal prism. The resemblance is made striking by the close approximation of the angles  $\alpha$  and  $\beta$  to the angles of the hexagonal prism ( $120^\circ$ ). The angle  $\alpha = 116^\circ 16'$  and  $\beta = 127^\circ 28'$ . Figure 262 is the reproduction of a photograph of such a trilling. The interpenetration twin of *chrysoberyl* (Fig. 255) also simulates the hexagonal symmetry.



## CHAPTER XIV.

### AMORPHOUS SUBSTANCES AND PSEUDOMORPHS.

**Amorphous Substances.**—Although the great majority of chemical compounds possess definite forms, there are some to which such forms seem to be entirely lacking. Bodies of this kind possess neither the geometrical properties of crystals nor do they have the physical properties peculiar to crystallized bodies. Their internal structure has not the regularity of that of crystals. Such substances are said to be *amorphous*, or they are described as *colloids*. Their crystallizing power is so weak that it is not capable of causing the molecules in which it resides to group themselves in accordance with the symmetry of any crystal system, except, perhaps, under the most favorable conditions. Under ordinary conditions this power is not exerted sufficiently to effect any result, and so the material is put together according to no definite plan, and consequently it possesses no definite external form. The shapes exhibited by such substances are the result of external conditions or of forces not inherent in the molecules of the substance. They are largely, if not entirely, accidental.

**Pseudomorphs.**—Idiomorphic forms of crystals are determined by the action of certain forces, called, for lack of a more definite name, crystallizing forces, which appear to be inherent in the substance of which the crystals are composed. The forms produced by their action are just as characteristic of the crystallized material as are its chemical reactions with other substances. Very frequently, however, substances are met with possessing definite crystal forms that are different from those which they usually possess, but which are similar to forms possessed by some other substance. The unusual forms originally belonging to some pre-existing substance and have been appropriated by the substance now possessing them. For instance, the mineral *limonite* ( $\text{Fe}_2\text{O}_3(\text{OH})_3$ ) is usually in globular or botryoidal forms (see Figs. 229 and 230). Sometimes, however, it occurs in cubes (Fig. 263).

which are known to be the forms in which pyrite ( $\text{FeS}_2$ ) crystallizes. *Bodies possessing forms borrowed from other substances are known as pseudomorphs* ( $\psi\epsilon\upsilon\delta\acute{\eta}\varsigma$ , false, and  $\mu\omicron\rho\phi\acute{\eta}$ , form). This term is applied not only to the form itself, but as well to the substance exhibiting it. In the latter sense a *pseudomorph* is a body possessing the form of one substance and the chemical and physical properties of another. Pseudomorphism is the assumption by one substance of the form of some other pre-existing one.

**Explanation of Pseudomorphism.**—The explanation of pseudomorphism is comparatively easy. A substance possessing its own distinctive form may be changed by the action of the



FIG. 263.—Pseudomorphs of limonite after pyrite. Pyrite crystal in the middle.

carbon dioxide, the oxygen, or the moisture of the atmosphere, or by some other agency into another substance differing from the original substance in nearly all of its morphological and physical properties. The material of the original substance is completely replaced by the new substance, but its external form remains unchanged. In such cases there results a pseudomorph.

The mineral *cuprite* ( $\text{CuO}$ ) often forms little octahedra. By exposure to the atmosphere cuprite changes to *malachite* ( $\text{CuCO}_3 \cdot \text{Cu(OH)}_2$ ), a monoclinic mineral crystallizing in acicular monoclinic crystals. When the change from the cuprite to the malachite takes place slowly, the former mineral is replaced, molecule for molecule, by the latter, the result being a mass of malachite with the outward form of the cuprite, or a pseudomorph of malachite after cuprite. The malachite is in reality monoclinic, as may be learned by examining it optically, but it possesses the shape of a regular crystal. In the same way we find *gypsum*

( $\text{CaSO}_4 + 2\text{H}_2\text{O}$ ) pseudomorphs after *anhydrite* ( $\text{CaSO}_4$ ), *limonite* ( $\text{Fe}_2\text{O}_3(\text{OH})_3$ ) pseudomorphs after *pyrite* ( $\text{FeS}_2$ ), etc. Petrified wood may likewise be described as a pseudomorph of *opal* ( $\text{SiO}_2 + \text{Aq}$ ) after wood.

**Paramorphs.**—Sometimes what is apparently the same chemical compound occurs in two different forms in nature—it is *dimorphous*, crystallizing in one form under certain conditions and in an entirely different form under other conditions. If, after the formation of crystals of such a substance, the conditions change, the entire mass of the crystal may pass over into the second form, producing a pseudomorph of the second substance after the first one. The external form of the second substance is now exactly similar to that of the first one, but its molecular structure is entirely different, thus giving rise to a genuine pseudomorph of a kind that has been distinguished by the name paramorph. *A paramorph is a pseudomorph of one form of a dimorphous body after the other form.*

One of the most familiar illustrations of a dimorphous substance is *sulphur*, which separates from a solution in carbon bisulphide as orthorhombic crystals, and from a molten mass as monoclinic needles. When allowed to stand at the normal temperature of the air, the monoclinic variety passes into the orthorhombic variety. The material still possesses the acicular form of the monoclinic sulphur, but its molecular structure is that of the orthorhombic variety. Here we have an example of a paramorph of orthorhombic sulphur after the monoclinic variety.

**Two Classes of Pseudomorphs.**—The processes described above as originating pseudomorphs are chemical, hence pseudomorphs produced by them are known as *chemical pseudomorphs*. There is another class of pseudomorphs, however, known as *mechanical pseudomorphs*. The forms of these bodies are not produced by the replacement of the substance of a crystal, particle by particle, by the pseudomorphing substance. They are produced simply by the filling of a mould left by the solution of some pre-existing crystal. The original crystal may become incrustated with some insoluble material. Its substance may then be dissolved, leaving a cavity of the shape of the crystal. If this



cavity be filled with a new substance, and then the enveloping material be removed, the new substance will necessarily possess the form of the original crystal.

At Girgenti, in Sicily, pseudomorphs of *calcite* ( $\text{CaCO}_3$ ) after *sulphur* are sometimes met with. The origin is explained as follows: The sulphur crystals were incrustated with a coating of *barite* ( $\text{BaSO}_4$ ). The temperature in the neighborhood rose until the sulphur melted and disappeared, leaving a mould of itself constructed of barite. By the infiltration of a solution containing calcium carbonate and the deposition of this substance as cal-

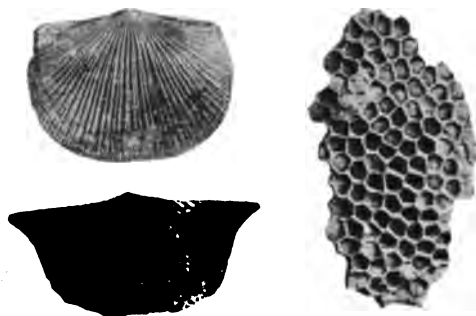


FIG. 264.—Fossils. Pseudomorphs of dolomite after Mollusks and Coral.

cite in the mould, the cavity was filled. Upon the removal of the barite coating a mass of calcite was left with the form of the sulphur crystals.

**Fossilization.**—Fossils are pseudomorphs of mineral substance after organisms or parts of organism. The processes of fossilization are exactly analogous to those of pseudomorphism. The original organism may have been replaced, molecule by molecule, with the fossilizing substance, or it may have been dissolved from the rock in which it was imbedded, leaving a cavity of its own shape, which afterward was filled with mineral substance. Fossils produced by the latter process preserve only the external form of the original organism, while those produced by replacement often retain even the minute internal structure of the original.

The fossilizing substance is usually calcite ( $\text{CaCO}_3$ ), dolomite ( $(\text{CaMg})\text{CO}_3$ ), or silica ( $\text{SiO}_2$ ) in some form, though fossils composed of other minerals are not uncommon (Fig. 264).

## CHAPTER XV.

### CRYSTAL PROJECTION.

**Projection.**—By the term projection in crystallography is meant the representation of crystals on a plane surface; i.e., on the surface that contains their figures. There are several graphic methods by which the planes on crystals are represented, among which are the linear projection and the spherical projection. These exhibit the relations of the planes to one another without reference to the shape of the crystal bounded by them. Other methods of projection—one of which is the clinographic projection—represent the crystal as it appears to the eye under certain conditions. This projection is a picture.

**The Linear Projection.**—This method of projection *represents each face on a crystal by its line of intersection with the plane on which the projection is made, when that face is assumed to pass through the unity of the vertical axis.* The plane taken as the plane of projection is usually that which includes the lateral axes of the crystal. The projection then appears as a number of straight lines that run in different directions across this plane and at different distances from the point representing the center of the crystal. Figure 265 shows the projection of the icositetrahedron.

The planes are all *assumed to pass through the unity on the vertical axis because the intercepts on this axis cannot be indicated directly in the projection.* They must therefore be indicated indirectly and this is done by imagining them moved parallel until they pass through the unity on  $c$ , and drawing their intersections with the plane of projection by joining the intercepts which they make on the lateral axes in this position. The planes thus projected are parallel to the planes on the crystal. The ratios of their intercepts are the same as they were before the imagined movement, and consequently, from the crystallographic

point of view, they are the same planes. Practically, the method consists in writing the intercepts of the plane to be projected in the form of a ratio and reducing to unity the term relating to the  $c$  axis. By laying off on the lines representing the lateral axes distances corresponding to the reduced intercepts on these axes and connecting them by a straight line, the projection of the plane is obtained.

In all the systems with three axes the lateral axes are represented in the projection by two lines perpendicular to each other. They are usually drawn dotted to distinguish them from

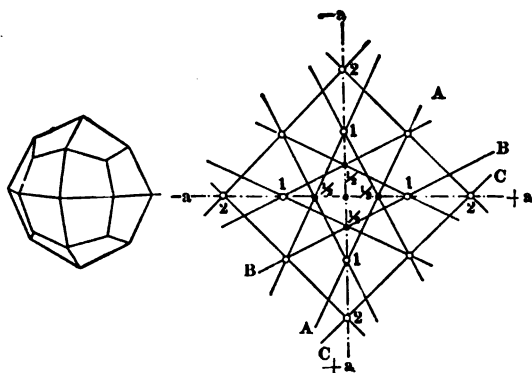


FIG. 265.—Form  $2O_2$  and its linear projection.

the projections of the planes which are drawn solid. In the hexagonal system the axes are represented by three lines intersecting at  $60^\circ$ . *In the systems with equal unities on the lateral axes equal distances are laid off on the projection of these axes to represent unity. In the other systems the distances that must be laid off to represent the unities must correspond to the ratio between the unities on  $a$  and  $b$  for the crystal to be projected.*

Usually only the upper half of the crystal is projected, as this projection in most cases represents the lower half as well. In the case of certain hemihedral and tetartohedral forms, in which the planes on the two halves are differently related to the axes, two projections must be made to represent the entire form. These, however, may be indicated in the same figure by designating the

upper and lower planes by some conventional sign. For instance, the projections of the upper planes may be drawn solid and those of the lower planes may be dotted.

Suppose the form  $2O_2$  is to be projected. The lateral axes are first indicated as two dotted lines perpendicular to one another (see Fig. 265). Convenient equal lengths are laid off as unities. The form consists of 12 planes above the plane of projection, three of which are in each octant. In the octant in which all the intercepts are positive the symbols of the planes are:

$$2a : b : 2c; a : 2b : 2c; 2a : 2b : c, \text{ or}$$

$a : 1/2b : c; 1/2a : b : c; 2a : 2b : c$ , when the intercept on  $c$  is reduced to unity.

For the projection of the first plane lay off on the right-left axis a distance equal to  $1/2$  the length decided upon as unity and connect this point by a straight line with the unity distance on

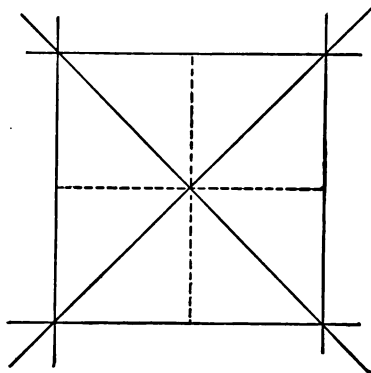


FIG. 266.—Linear projection of dodecahedron.

the front-back axis ( $A-A$ ). For the projection of the second plane lay off  $1/2$  the unity distance on the  $a$  axis and connect this by a straight line with the unity point on  $b$  ( $B-B$ ). For the projection of the third plane lay off two unities on  $a$  and  $b$  and connect by a straight line ( $C-C$ ). By a similar process the projection of the planes in the other three upper octants are drawn and the figure is completed (see Fig. 265).

Planes that are parallel to the vertical axis must necessarily pass through the intersection of the axes in the projection, since,

when such planes are made to pass through the unity on  $c$ , they cut this axis throughout its entire length. Thus the projection of  $\infty O$  consists of two lines passing through the unity distances on  $a$  and parallel to  $b$ , two lines passing through the

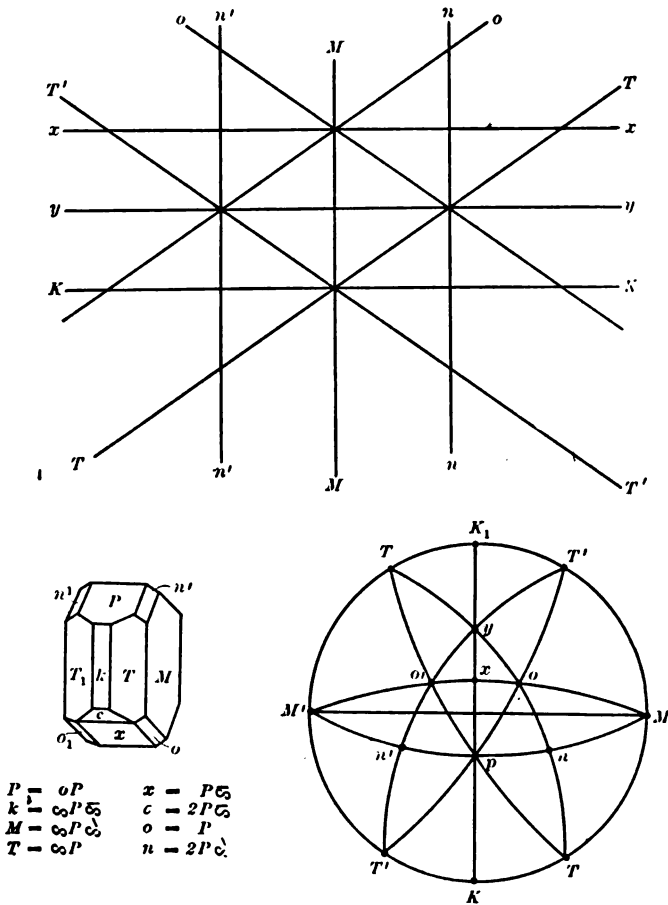


FIG. 267.—Crystal of orthoclase with linear and spherical projections.

unities on  $b$  and parallel to  $a$ , and two lines passing through the center of the projection and bisecting the angles between the axes. The symbol of the first two lines is  $\infty b : c$ ; of the second pair,  $\infty a : b : c$ ; and of the third pair  $a : b : \infty c$ . (See Fig. 266.)



**Spherical Projection.**—In the spherical projection the position of each plane on the upper half of the crystal is represented by the *point of intersection of a perpendicular to the center of its face with the surface of a hemisphere at whose center the crystal is supposed to be.* The hemisphere is then projected on a plane passing through its equator. The projection appears as a lot of dots arranged with the same symmetry within a circle as that of the planes on the crystal. Figure 267 shows the projection of a crystal of orthoclase (a monoclinic mineral) by the linear and the spherical methods.

**Crystal Drawing.**—The representation of crystals as they appear in nature (clinographic projection), or the drawing of crystal figures such as are used in this book, is different from ordinary perspective drawing in that lines which are parallel on the crystal are made parallel in the representation.

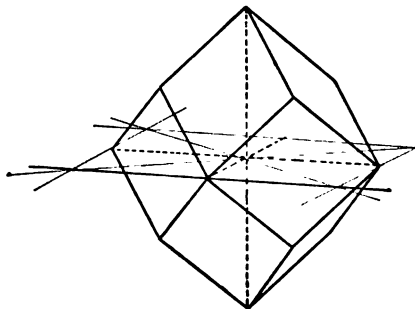


FIG. 268.—Illustration of method of drawing the dodecahedron.

In constructing a crystal drawing the axes are first represented as they would appear if the eye were viewing them from the right and from above. A linear projection is then made on the lateral axes, and lines representing the directions of the interfacial edges between contiguous planes are drawn from the ends of the vertical axes to the points in the projection where the lines representing the planes intersect.

Figure 268 shows the projection of the axes in the regular system, with the projection of the planes of the dodecahedron ( $\infty O$ ) upon the plane of the lateral axes, and the completed drawing made from this projection.

**Projection of the Crystal Axes.**—The most important step in the drawing of any crystal is the projection of its axes, because these serve as the foundation upon which the drawing is constructed. After the preparation of the perspective view of the axes the completion of the drawing is comparatively easy.

Different methods of making the projection of the axes are employed, the choice between them depending upon those features of the crystal that it is desired to emphasize. For general purposes the vertical axis is represented as a vertical line, and the lateral axes are drawn in perspective on the assumption that they are viewed by the eye at a certain angular distance ( $\delta$ ) to the right of the center of the crystal and elevated a certain angle ( $\epsilon$ ) above it. Different values may be assigned to  $\delta$  and  $\epsilon$ , and as a result different views of the crystal may be shown. It is

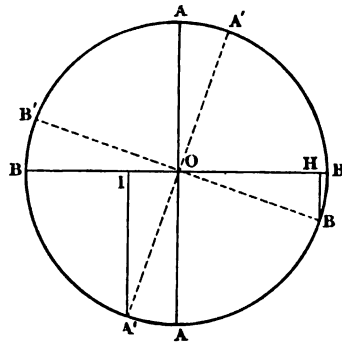


FIG. 269.

usual, however, to assume such values as may be expressed by a simple ratio between equal axes after projection. If the ratio between the two axes  $OI$  and  $OK'$  (Fig. 270) is  $1 : 3$  and the ratio between  $AI$  and  $OI$  is  $1 : 2$ , then  $\delta = 18^\circ 26'$  and  $\epsilon = 9^\circ 28'$ .

Figure 269 represents the normal position of the lateral axes in the isometric or tetragonal system as viewed from the top of the crystal; i.e., along the direction of the vertical axis,  $c$ . If viewed from the direction of the axis  $AA(a)$  this will appear as a point, while  $BB(b)$  will appear in its true length.

After revolution to the left through the angle  $AOA'$  ( $=\delta = 18^\circ 26'$ ) the axis  $AA$  will have the position  $A'A'$ ; i.e., its front half will be lengthened to  $OI$ , and  $BB$  will assume the position  $B'B'$ ; i.e., its right half will be shortened to  $OH$ .

If, now, the eye be elevated (at the angle  $\epsilon$ ) the lines  $A'I$ ,  $AO$ , and  $B'H$  will be projected below  $I$ ,  $O$ , and  $H$  to distances proportional to the lengths of the respective lines. If  $\delta = 18^\circ 26'$  and

$\epsilon = 9^\circ 28'$ , then  $OI : OH = 1 : 3$  and  $a'I : OI$  as  $1 : 2$  (in which  $a'I$  is the projection of  $AI$  below  $I$ ).

**Projection of the Axes for the Isometric System.**—If the values for  $\delta$  and  $\epsilon$  be assumed as above, then the method of construction of the isometric axes is as follows: Draw two lines  $LL'$

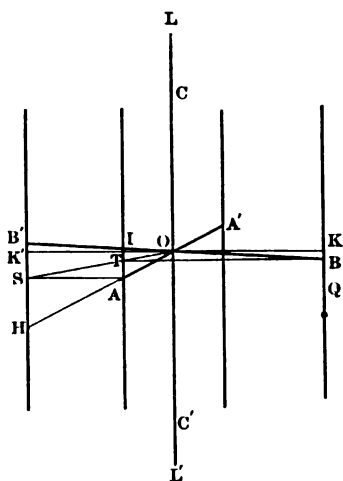


FIG. 270.—Construction of isometric axes.

$LL'$  and  $KK'$  at right angles to one another (Fig. 270). Make  $KO = K'O = \text{unity on } b$ , and divide  $KK'$  into three equal parts. Draw verticals through the four points thus obtained on  $KK'$ , and below  $K'$  lay off  $K'H = \frac{1}{2} K'O$ . Draw  $HO$ , which will give the direction of the front lateral axis. Its length will be that portion of this line included between the two inner verticals,  $A$  and  $A'$ .

Draw  $AS$  parallel to  $K'O$  and connect the points  $S$  and  $O$ . From the intersection of this line with the inner vertical,  $T$ , draw

$TB$  parallel to  $K'K$ . From point,  $B$ , thus obtained draw the line  $BB'$  through  $O$ . This will be the second lateral axis,  $a$ .

Below  $K$ , lay off  $KQ = \frac{1}{3} OK$  and make  $OC = OC' = OQ$ ; then  $CC'$  will be the length of the vertical axis.\*

**Projection of the Axes for the Tetragonal and Orthorhombic Systems.**—The axes constructed for the isometric system may be readily adapted to both the other systems with rectangular axes by merely laying off portions of the lines  $AA'$  and

\* In order to avoid the necessity of projecting the axes each time a drawing is to be made, it is advisable to construct a set on a piece of cardboard and prick holes at the ends of the axes and their point of intersection. The axes can then be transferred to drawing-paper by making dots through these holes and connecting them by straight lines. The relative lengths of equal unity distances will be indicated by the positions of the dots. Longer or shorter unity distances are secured by increasing or diminishing proportionately the lengths thus obtained. It is important that the construction be made with all possible accuracy, otherwise the completed figures may be distorted. This is secured in part by making the original drawing of such a size (*e.g.*, by making the entire length of the  $b$  axis = 4 inches) that small errors will be practically eliminated when the lengths of the axes are reduced to the dimensions ordinarily employed in drawing crystals.

$CC'$ , which are proportional to the lengths expressed in the axial ratios of the crystals to be figured.

For instance, if the axial ratio of the crystal to be drawn is  $a : b : c = 1.5 : 1 : 2.4$ , proceed as follows: Transfer the permanent projection of the axes to the paper upon which the drawing is to be made. Take proportional lengths of the axes as thus constructed if more convenient than the entire lengths. These distances will represent ratios of  $1 : 1 : 1$  on the three axes. Increase the length on  $a$  by .5 of *itself* and that on  $c$  by 1.4 of *itself*. The resulting lengths will have the relations  $1.5 : 1 : 2.4$ , or the axial ratio desired. Indicate these by dots and treat them as the unity intersections on the several axes.

In the case of a tetragonal crystal like *zircon*, the axial ratio of which is  $a : c :: 1 : .64$ , the two lateral axes remain unchanged, while the vertical axis must be made .64 of the length  $CC'$ .

For an orthorhombic crystal the axis  $BB'$  alone remains unchanged, while  $AA'$  and  $CC'$  are both changed to the proportionate lengths belonging to the substance in question.

**Projection of the Monoclinic Axes.**—To project the inclination,  $\beta$ , of the clinoaxis, construct the axes as in the isometric system, and then lay off  $Oc = OC \cdot \cos \beta$ , and on  $OA'$  lay off  $Oa = OA' \cdot \sin \beta$  (Fig. 271). From  $c$  draw a line parallel to  $OA'$ , and from  $a$  another parallel to  $OC$ . From their intersection, a line ( $DD'$ ) drawn through  $O$  will give the direction of the clinoaxis. The directions of the other two axes remain unchanged. The relative lengths of the axes must now be laid off, according to the axial ratio of the substance, as in the orthorhombic system.

**Projection of the Triclinic Axes.**—In this case all three axes of reference intersect obliquely  $b \wedge c = \alpha$ ,  $a \wedge c = \beta$ ,  $a \wedge b = \gamma$ . If we start with the isometric axes, the first step in their adaptation

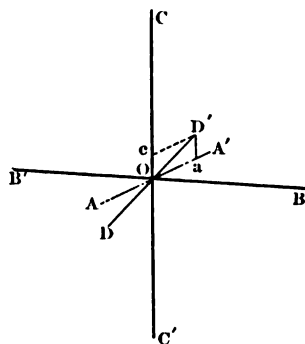


FIG. 271.—Construction of monoclinic axes.

to the triclinic system is to obtain the direction of the two vertical axial planes or pinacoids. To do this, lay off (Fig. 272) on OB

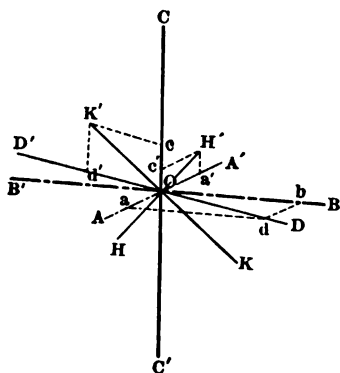


FIG. 272.—Construction of triclinic axes.

$Ob = OB \cdot \sin \phi$  ( $\phi$  being the angle  $\infty P \infty \wedge \infty P \infty$ , which is evidently not the same as  $\gamma$ ), and on OA,  $Oa = OA \cdot \cos \phi$ . The line drawn from the angle  $d$  of the parallelogram  $adbO$  through O will give the direction of the macropinacoidal section,  $DD'$ . To obtain the direction of the macroaxis ( $b$ ), lay off on  $OD'$ ,  $Od' = OD \cdot \sin \alpha$ ; and on  $OC$ ,  $Oc = OC \cdot \cos \alpha$ . From the parallelogram,  $d'OcK'$ , thus obtained, the diagonal,  $K'K$ , gives the macroaxis. In a similar manner, the brachyaxis ( $a$ ),  $HH'$ , is found by laying off on  $OA'$ ,  $Oa' = OA \cdot \sin \beta$ ; and upon  $OC$ ,  $Oc' = OC \cdot \cos \beta$ . The vertical axis  $CC'$  and the lateral axes  $HH'$  and  $KK'$  thus obtained are the axes of a triclinic crystal in which  $a : b : c = 1 : 1 : 1$ . Their relative lengths must now be given them in accordance with the axial ratio of the substance, just as in the orthorhombic and monoclinic systems.

### Projection of the Hexagonal Axes.

Construct an orthorhombic set of axes whose axial ratio,  $a : b : c$ , is  $\sqrt{3}$  ( $= 1.732$ ) : 1 :  $c$  ( $c$  being given the value of the vertical axis belonging to the substance to be drawn) (Fig. 273); connect the extremities of the two lateral axes, and, in the rhomb thus formed, the obtuse angles, at the ends

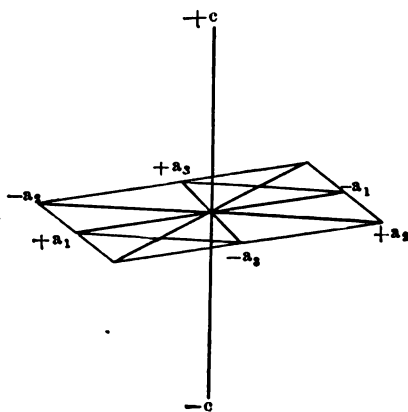


FIG. 273.—Construction of hexagonal axes.

of the  $b$  axis, will be exactly  $120^\circ$ . If lines be now drawn parallel to  $b$ , through points on the axis  $a$ , half way between its extremities and the center  $o$ , the rhomb will be converted into a hexagon, with all of its angles exactly  $120^\circ$ . If we connect the diagonally opposite angles of this hexagon, we shall obtain the projection of the hexagonal axes required.

**Construction of the Drawing.**—Having made a projection of the axes the next step is to transfer to the lateral axes the plane projection of the faces on the crystal to be drawn (see Fig. 268). Remembering that all the lines in this projection represent the traces of the planes when they are supposed to pass through the unity point on  $c$ , it follows that this point is common to all the planes represented in the projection.

The outline of the crystal is shown by drawing the interfacial edges between neighboring planes. These edges are represented as lines in the drawing. It is only necessary, then, to find one point other than the unity on  $c$  which is common to the two planes whose intersection is desired. This is the point of intersection of the two lines representing the planes in the projection. Thus we obtain two points which are at the same time in the two intersecting planes. The line joining them gives the direction of their interfacial edge. If, therefore, we draw a line from the point representing the intersection of the two planes in the projection to the unity point on  $c$  we have the direction of the interfacial edge of these planes. In cases when the intersecting planes are represented by parallel lines in the projection, their interfacial edge is indicated in the drawing by a line parallel to these lines in the projection.

It will be noted that the lines drawn by this procedure indicate only the directions of the desired intersections. They all diverge from the unity point on the vertical axes. In the crystal the intersections are not so distributed. They are lines joining certain points on the crystal surface which may or may not be at the unity point on the  $c$  axis. In constructing the drawing the lines with their proper directions must be drawn from points that have the relative positions of the corresponding points on the crystal.

Usually a prominent face is first outlined by drawing the

proper lines. Then from points on the outline of the face other lines are drawn to represent the interfacial edges that extend from it, and so on until the complete figure is produced. The size of the figure will be determined by the size of the first face outlined. Its proportion will in some cases be fixed by the symmetrical development of the figure. In most cases, however, the proportions must be controlled by noting the proportional lengths of the different interfacial edges on the crystal and making them correspond in the figure. In other words, the direction of the lines representing the interfacial edges are fixed by the projection, but the habits of the crystals represented are indicated by the relative sizes of the planes in the drawing or the relative lengths of the lines representing their interfacial edges.

**PART II.**  
**PHYSICAL CRYSTALLOGRAPHY.**





## CHAPTER XVI.

### INTRODUCTION: PHYSICAL SYMMETRY AND PHYSICAL AGENCIES.

**Physical Symmetry.**—The material of which crystals are composed, like all other matter existing throughout the universe, is made known to our senses through its properties. The substance of crystals obeys the same laws of physics that govern all other matter, but these laws manifest themselves a little differently than they do in non-crystallized bodies, because of the fact that the crystal particles are put together in a definite manner.

In amorphous bodies physical forces produce exactly similar effects when acting in different directions, provided the bodies are homogeneous throughout. In crystalline bodies the case is different. In these the effects of forces vary with directions, and the variation is in the closest accord with the symmetry of the substance acted upon; i.e., with the arrangement of its component particles as expressed in the symmetry of its idiomorphic forms. *Every geometrical plane of symmetry is also a physical one, and all geometrically equivalent directions are also equivalent with respect to physical agents.* When any given force acts upon a crystal it will produce similar effects along parallel directions, and effects varying in degree along different directions which are not symmetrically disposed about planes and axes of symmetry.

Of course a force like the force of gravity may act upon a crystal as a whole, and then the effect of symmetry will not be apparent. In all cases, however, where the force acts successively upon different particles of the crystal or when it is transmitted by these particles, the effect of symmetry is clearly noticeable.

When considering the effects of forces acting upon the surfaces of crystals it must be kept in mind that the symmetry of the planes bounding a crystal is something quite different from the symmetry of the crystal itself. Only those crystallographic planes of

symmetry that are perpendicular to the bounding planes can be planes of symmetry for these surfaces. It is impossible to conceive of a plane surface symmetrically disposed about a plane inclined to it.

As an illustration of the relation existing between the symmetry of planes and that of the crystals on which they occur, let us consider the basal planes on holohedral forms belonging to the different crystallographic systems (see Fig. 274). The top plane of the cube in the isometric system corresponds to the basal pinacoid in the other systems. In the isometric system this plane is symmetrical with respect to the four planes inclined to one another at angles of  $45^\circ$  and intersecting in a common line, which is the

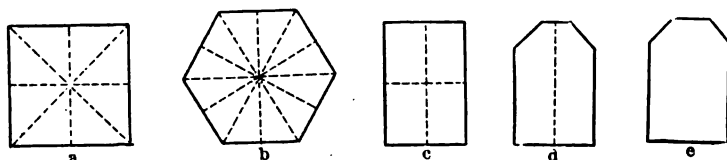


FIG. 274.—Diagrams illustrating symmetry of planes at terminations of the vertical axes in holohedrons of the different systems.

vertical axis of the cube (a). In the hexagonal system the basal plane is symmetrical with respect to six similarly intersecting planes inclined to one another at angles of  $60^\circ$  (b). The corresponding face in the tetragonal system is similar to the cubic face in its symmetry. In the orthorhombic system it is symmetrical with respect to two planes of symmetry perpendicular to each other (c). In the monoclinic system the basal plane is traversed by a single plane of symmetry (d), and in the triclinic system it is crossed by no symmetry plane (e).

Forces acting upon the faces of a crystal will be governed in their effects by the symmetry of the faces. From the symmetry of the effects produced the symmetry of the faces is disclosed and from the symmetry of several faces the symmetry of the whole form to which they belong may be deduced. Consequently, the grade of symmetry possessed by a substance may be discovered even when complete crystals are not obtainable.

**Physical Agencies.**—The physical forces producing effects

that are of the greatest importance in determining the grade of symmetry of crystals may be classified as mechanical, optical, thermal, and electrical. The nature of these forces is a subject dealt with in physics. At present we are more directly concerned with the character and distribution of the effects they produce. These effects, as has already been stated, accord in their general character with the symmetry of the crystal upon which they act.

The relations of the substance toward the forces are known as their properties. Thus we speak of the optical properties of crystals when we refer to their action with reference to the forces that produce light, of their thermal properties when we refer to their action under the influence of thermal forces, etc.

## CHAPTER XVII.

### MECHANICAL PROPERTIES OF CRYSTALS.

**Mechanical Properties.**—The mechanical properties of crystals are those they exhibit with respect to mechanical forces. They are discussed under the following heads: Elasticity, tenacity, cohesion, cleavage, fracture, hardness, and density.

**Elasticity.**—By elasticity is meant that property which causes bodies to resist forces tending to change their form. This power of resistance varies in different substances and varies along different directions in the same body, provided it is not amorphous. It is expressed by the coefficient of elasticity, which is the relation existing between the length of a standard-sized bar of the substance and the elongation it suffers under the influence of a given pull. *For all amorphous bodies the coefficient of elasticity is equal in all directions;* that is, bars of equal size cut from amorphous bodies in any direction will be equally elongated when subjected to the same strain.

*In crystals the value of the coefficient of elasticity varies according to the direction in which the stress is applied.* In isometric crystals it is equal in directions parallel to the three crystallographic axes, and varies from this in other directions always being equal, however, in parallel directions, and in directions that are symmetrical with respect to one another.

The coefficient of elasticity in salt ( $\text{NaCl}$ ), for instance, is as 1 to .7 in rods cut perpendicular to the cubic face and those cut perpendicular to the octahedral face.

Although the symmetry of a crystallized body may be determined by a study of its elastic properties even when crystals of it are not obtainable, nevertheless, since there are much more convenient methods than this that may be employed for the purpose, the elastic properties are not made use of to any great extent.

**Tenacity.**—If the elasticity of a body is defined as the quan-

tity of resistance it opposes to deforming influences, we may define tenacity as the quality of this resistance.

With respect to tenacity, substances are distinguished as *brittle*, *sectile*, *malleable*, *flexible*, and *elastic*. A brittle substance is one that breaks into powder when cut with a knife, as does *calcite* ( $\text{CaCO}_3$ ). A sectile substance may be cut, but it pulverizes under blows, as, for instance, *gypsum* ( $\text{CaSO}_4 + 2\text{H}_2\text{O}$ ). A malleable substance flattens when hammered upon, as *copper* and other metals. A flexible substance will bend when subjected to forces properly applied, and will remain bent when the action of the



FIG. 275.—Cleavage piece of calcite showing cleavage cracks.

forces ceases, as *talc* ( $\text{H}_2\text{Mg}_3(\text{SiO}_3)_4$ ) and *asbestos* ( $(\text{CaMg})\text{SiO}_3$ ). An elastic substance will fly back into its original position when the force that bends it is removed, as *mica* ( $\text{H}_2(\text{KNa})\text{Al}_3(\text{SiO}_4)_3$ ).

**Cohesion.**—Cohesion is the resistance offered by bodies to the separation of their particles. Those special characters that are dependent upon the strength of this resistance are cleavage, fracture, and hardness.

**Cleavage.**—Many crystals possess a marked tendency to split along certain directions in preference to others, in consequence of differences in cohesive power in different directions. The planes along which such splitting occurs are known as *cleavage planes* (Fig. 275). They must be perpendicular to the direction of minimum cohesion and their perfection must depend upon the dif-

ference in the cohesive force along different directions—the greater the cohesion difference the better the cleavage.

The cleavage planes are always parallel to planes that are crystallographically possible; that is, they are planes with rational indices. Moreover, *if one crystal face is parallelized by a cleavage plane, all other faces belonging to the same crystal form are also parallelized by cleavage planes, and along all of these planes the cleavage is equally easy.* This must be so because cohesion is a property of the molecules and in crystals the molecules are regularly arranged. Their geometrical forms and physical properties are but different expressions of this arrangement. *When two different cleavages are present, i.e., when a crystal cleaves parallel to faces belonging to different crystal forms, the ease with which the cleavages are produced is unequal, and the character of the surfaces produced differs.* In *galena* (PbS), for instance, cleavage is equally easy in three directions perpendicular to one another; i.e., in directions parallel to cubic faces—the cleavage is cubical. In *sphalerite* (isometric ZnS) the cleavage is equal along planes parallel to dodecahedral faces—it is dodecahedral. In *barite* (orthorhombic BaSO<sub>4</sub>), on the other hand, there are two unequal cleavages. The most easy one is parallel to oP and the most difficult one parallel to P.

The differences in the character of cleavages produced in a substance often serves to determine its system of crystallization. *Anhydrite* (CaSO<sub>4</sub>), for instance, cleaves along three planes perpendicular to one another, but with different degrees of perfection. The differences in ease with which the cleavage takes place along the different planes serves to fix the symmetry of the substance as orthorhombic.

According to the ease with which cleavage is effected and the evenness of the surfaces produced by the cleavage, this is said to be *very perfect, perfect, distinct, indistinct, imperfect, interrupted, or difficult.*

The relative ease with which cleavage is produced along different planes is measured by cutting rods of a substance with their long axes parallel to different crystallographic lines, and then straining them until they break. The symmetry of the cleavage

may be determined by measuring the force of the strain under which the cleavage is effected in a number of rods.

**Gliding.**—Many crystals possess a property which is analogous to shearing, but which differs from this in the fact that it occurs only along certain planes, which are perpendicular to the direction of maximum cohesion—and which are known as *gliding planes*. Small portions of the crystals may be moved along the gliding planes without being separated from the unmoved

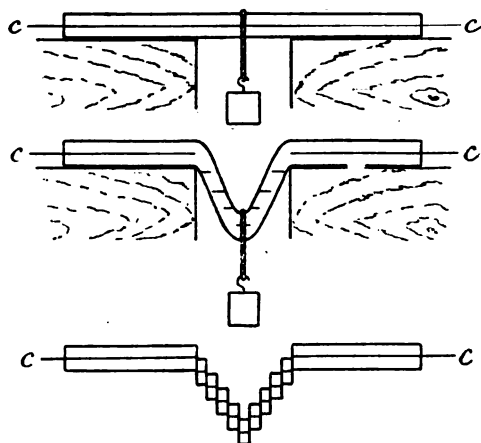


FIG. 276.—Diagrams illustrating shearing of ice. (After Linck.)

- A. Rod cut from ice crystal parallel to its vertical axis ( $cc$ ).  
 B. The same rod after being bent by loading its center. The vertical axis in the bent portion remains parallel to its original position.  
 C. Explanation of method of bending. Thin slices of the ice shear downward.

parts. In many instances the movement consists of a slipping along a series of parallel planes and is unlimited in amount, thus resembling true shearing. In other cases the movement is caused by a rotation of the molecules in a series of rows parallel to the gliding plane. Each molecule moves but a slight amount with reference to neighboring molecules, and the limit of movement is reached when the part of the crystal that has been deformed occupies the twinned relation to the undeformed part with the gliding plane as the twinning plane.

The two types of gliding are well illustrated by ice and calcite (Fig. 276). If a bar cut from a crystal of ice parallel to its vertical



axis be placed in a horizontal position and supported at its ends while weighted at its center, the middle portion of the bar will slowly sag downward. Examination of the bent portion will show that the sagging is not due to bending, as the crystallographic axis will be found to have suffered no deformation in the sagged portion of the bar. It will still remain parallel to its original

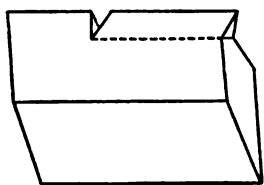


FIG. 277.—Artificial twin of calcite produced by gliding.

position. The sagging must therefore be due to downward slipping of a large number of parallel lamellæ (see Fig. 276).

If, on the other hand, a sharp knife-blade is placed perpendicular against the blunt edge of a cleavage rhomb of calcite and is gently forced into its substance, a slice of the material will move without fracturing until it assumes a twinned position with reference to the rest of the calcite (see Fig. 277). The corresponding axes in the moved portion will no longer be parallel to their original positions, but they will be rotated into twinned positions. The plane between that portion of the calcite that has been moved and that portion which has not been disturbed is the gliding plane.

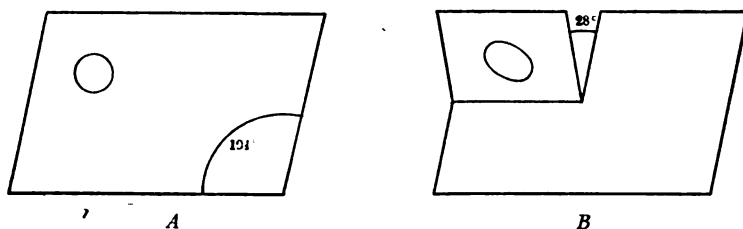


FIG. 278.—Diagrams illustrating process of gliding in calcite. The circle drawn near the upper corner (A) is elongated into an ellipse after gliding (B).

It is also the twinning plane. The fact that the horizontal rows of molecules have rotated is shown by the change of circles into ellipses during the movement (see Fig. 278).

**Secondary Twinning.**—Very frequently gliding is produced in opposite directions along a succession of parallel planes so that there results a series of lamellæ each in the twinning position

with respect to its neighbors. Polysynthetic twinning produced in this way is known as secondary twinning because brought about after the crystals exhibiting it were formed. It is a common phenomenon in the calcite grains composing marble (see Fig. 279).

**Percussion Figures.**—When a hard point is placed against the face of a crystal and then is tapped with a sharp stroke, cracks may be produced forming a star-like figure, the shape of which



FIG. 279.—Thin section of marble viewed in polarized light. The dark bars are secondary twinning lamellæ due to gliding under the influence of pressure. Magnified about 5 diameters.

is characteristic for many substances. The cracks are partings along definite crystal planes, and are closely related to the gliding planes.

The percussion figure on the cubic face of halite ( $\text{NaCl}$ ) is a four-rayed star with the rays parallel to the diagonals of the cubic face. The rays are cracks that are perpendicular to the face and therefore parallel to the four dodecahedral planes that truncate the vertical edges of the cube. On the octahedral face the percussion figure is three-rayed.

A six-rayed star is produced in a similar manner on cleavage pieces of mica (Fig. 280) which crystallizes in the monoclinic system. The ray which is parallel to the plane of symmetry—the clinopinacoid—is larger than the others, and is called the

characteristic ray. By its means the position of the clinopinacoid may be determined in plates of mica that show no crystal planes.

**Pressure Figures.**—If a very thin cleavage plate of mica is placed on a yielding support and pressed by a blunt point, another figure is produced which is also six-rayed when perfect (Fig. 281). Usually, however, some of the rays are missing and the figures consist of three or four cracks only. The rays of the pressure figure are perpendicular to those of the percussion figure.

Cracks having the directions of the rays of the pressure figure are often observed in pieces of mica and sometimes triangular

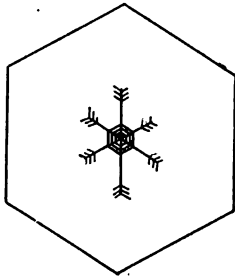


FIG. 280.—Percussion figure on basal plane of muscovite. The mineral is monoclinic and the long ray is parallel to the plane of symmetry,  $\infty P\infty$ .

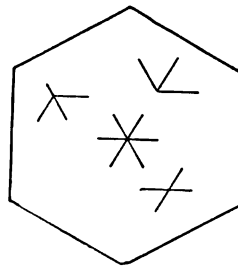


FIG. 281.—Pressure figure in basal plane of muscovite. The rays bisect the angles between those of the percussion figure.

fragments of the mineral are found in which actual separation has occurred along these directions (Fig. 282). The phenomena are due to pressure exerted on the crystal substance while in the rocks.

**Parting.**—Regular breaking along planes which are not cleavage planes is known as *parting*. It differs from cleavage in that it occurs only in certain places; i.e., along the cracks produced by pressure, along the planes separating twinned lamellæ, etc., while cleavage may take place equally well anywhere parallel to the cleavage plane (see Fig. 282).

**Fracture.**—When a force breaks a crystal in a direction which is neither a cleavage nor a gliding plane, or produces a break in an amorphous body, the separation takes place in an irregular way.

This kind of breaking is known as *fracture*. It is described according to the character of the surfaces produced, as *even*, *sp'in'ery*, *earthy*, *hack'y*, or *conchoidal*. A hackly fracture leaves surfaces that are ragged and rough, such as is exhibited by a broken piece of malleable metal. A conchoidal fracture leaves surfaces marked by concentric, or nearly concentric, curved lines, like the lines on many shells of molluscs (see Fig. 283). This kind of fracture is best exhibited by glass. The names applied to the other kinds of fracture are self-descriptive.

**Hardness.**—The hardness of a substance may be measured in a number of different ways, but the results obtained are not comparable. Consequently, a satisfactory definition of hardness is not yet possible.

A harder substance will scratch a softer one. The miner-



FIG. 282.—Fragment of muscovite parted along the planes of the pressure figure. (After Linck.)



FIG. 283.—Conchoidal fracture in obsidian.

alogist Mohs proposed the names of the following ten minerals to serve as a scale to which to refer all other minerals with respect to hardness. The scale begins with a soft mineral and ends with the hardest substance known.

## MOHS'S SCALE OF HARDNESS.

- |  |   |
|--|---|
| 1—Talc ( $\text{H}_2\text{Mg}_3(\text{SiO}_3)_4$ ) | 6—Feldspar ( $\text{KAlSi}_3\text{O}_8$ )       |
| 2—Gypsum ( $\text{CaSO}_4 + 2\text{H}_2\text{O}$ ) | 7—Quartz ( $\text{SiO}_2$ )                     |
| 3—Calcite ( $\text{CaCO}_3$ )                      | 8—Topaz ( $\text{Al}_2\text{F}_2\text{SiO}_5$ ) |
| 4—Fluorite ( $\text{CaF}_2$ )                      | 9—Corundum ( $\text{Al}_2\text{O}_3$ )          |
| 5—Apatite ( $\text{Ca}_5(\text{PO}_4)_3\text{F}$ ) | 10—Diamond (C).                                 |

A mineral that neither scratches any given mineral in the scale of hardness nor is scratched by it is said to have the same hardness as this; if it scratches one of the scale minerals and is scratched by the next harder one, its hardness is between that of the former and that of the latter. For example, a mineral that neither scratches quartz nor is scratched by it has a hardness of 7. One that scratches feldspar and is scratched by quartz possesses a degree of hardness between 6 and 7.

Minerals with a hardness of 2 or under can be scratched with the finger-nail. Those whose hardness is between 2 and 4 can be scratched easily with the point of a knife. Those with a hardness of 4 to 5 cannot be scratched with a knife, but can easily be scratched with a good file. Only those minerals whose hardness is greater than 5 will scratch window-glass. Only those with a hardness above 7 will strike fire with steel.

The degrees in the Mohs's scale are entirely arbitrary and the intervals between them are very unequal. The following table records the relative hardness of the minerals comprising the Mohs scale as determined by different men using different methods:

RELATIVE HARDNESS OF CERTAIN MINERALS AS DETERMINED BY  
DIFFERENT INVESTIGATORS.

Mohs	1	2	3	4	5	6	7	8	9	10
Franz.....			13.5	54	135	390	670	340	1000	
Pfaff.....	1.13	12.03	15.3	37.3	53.5	191	254	459	1000	
Rosival.....	.33	1.25	4.5	5	6.5	37	1120	175	1000	140,000
Jaggar.....		.04	.26	.75	1.23	25	40	152	1000	

**Usual Method of Determining Hardness.**—The relative hardness of different substances is usually determined by measuring the force required to scratch their smooth surfaces. This is done by means of an instrument known as a *sklerometer*. One form consists of a sharp point of steel or diamond capable of being weighted and a movable platform that can be drawn beneath it. A plate of the substance to be tested is placed on the

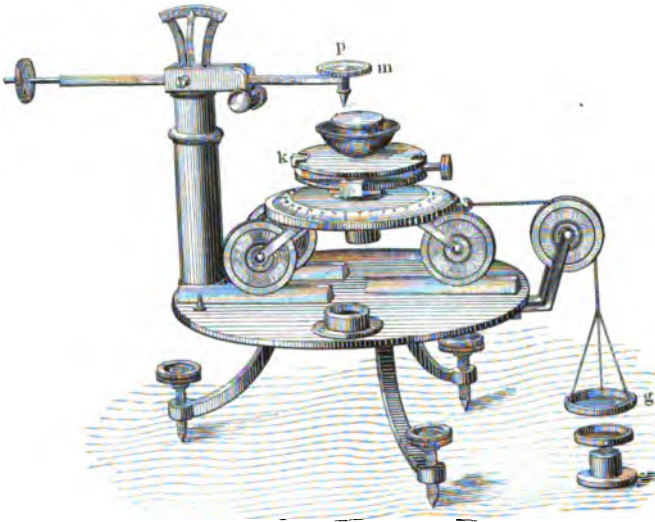


FIG. 284.—Sklerometer—an instrument for determining differences in hardness in different directions on crystal faces.

platform and the point pressed down upon it. The force necessary to drive the point sufficiently deep into the substance to leave on it a scratch when the platform is moved is a measure of its hardness. One form of the sklerometer is shown in the accompanying figure (Fig. 284). Its essential features are the little car or movable platform (*k*), the point (*m*), the platform for the load (*p*), and the basket (*g*), containing a constant weight by which the car is drawn beneath the point.

**Differences in Hardness.**—As the result of experimentation with the sklerometer it is learned that not only do different substances possess different degrees of hardness, but also that *the*

*same substance when crystallized offers greater opposition to a scratching agent acting in a certain direction than to the same agent acting along some other direction.*

The triclinic mineral *cyanite* ( $\text{Al}_2\text{SiO}_5$ ), for instance, resists scratching parallel to  $c$  much less effectually than scratching at right angles to this direction. Cyanite is harder in directions perpendicular to  $c$  than in directions parallel to this axis. In general it is found that *crystals are equally hard in directions that are symmetrical with respect to each other, and that the hardness differs in directions that are not symmetrical.*

The differences in hardness in any crystal seem to be governed largely by the cleavage, scratches always being produced more easily parallel to the cleavage than across it. Since the cleavage is in accord with the symmetry of the crystal, its hardness must likewise be symmetrical. Moreover, if the cleavage is inclined to the surface under investigation the hardness varies in opposite directions. In figure 285, for instance, the hardness is greater in the direction b to a than in the direction a to b.



FIG. 285.

**Curves of Hardness.**—A curve drawn on a plane surface in such a way as to express the differences in hardness exhibited by a crystal face in different directions is known as a curve of hardness. The relative hardness in different directions is represented by the relative lengths of straight lines passing through the center of the curve and terminating at both ends in its circumference.

Figures 286 and 287 are the curves of hardness for the cubic faces of crystals of *halite* ( $\text{NaCl}$ ) and of *fluorite* ( $\text{CaF}_2$ ), respectively. In the first figure the lines along the diagonals of the faces are longer than those parallel to their edges. This indicates that the hardness in the former direction is greater than that in the latter. The hardness along intermediate directions is intermediate in degree. In the case of fluorite the hardness is least in directions parallel to the diagonals of the faces and greatest in directions perpendicular to the cubic edges.

The symmetry of curves of hardness always corresponds to the

symmetry of the faces for which they are constructed. Consequently from the differences in hardness exhibited by substances the symmetry of their crystals may be determined.

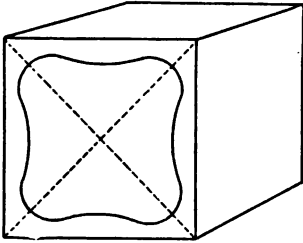


FIG. 286.

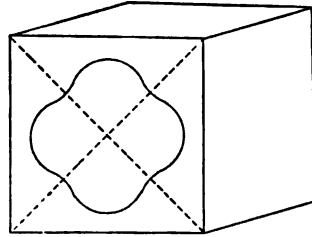


FIG. 287.

Curves of hardness in cubic faces of halite (Fig. 286) and fluorite (Fig. 287.)

Figure 288 shows the curve of hardness as determined by three observers on the basal plane of *calcite*, which is an hexagonal mineral crystallizing in rhombohedral hemihedral forms. Three planes of symmetry are perpendicular to the basal plane and

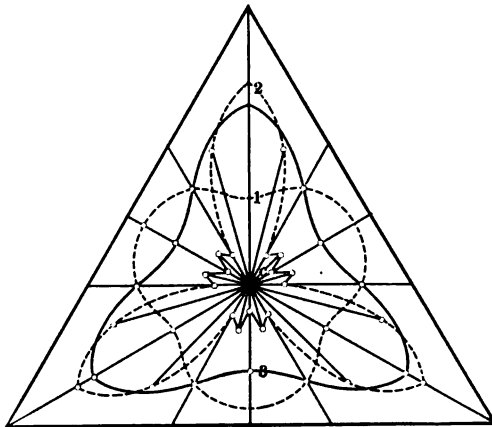


FIG. 288.—Curves of hardness on the basal plane of calcite as determined by three different investigators. Although varying in shape they all exhibit the same symmetry. (After Müller).

these intersect one another at angles of  $120^\circ$ . In the figure their positions are indicated by the lines drawn from the angles of the circumscribing triangle to the centers of the opposite sides.



The three curves, though of different shapes, are all symmetrical with respect to these three lines. Had the mineral been holohedral the curves would have been symmetrical about six lines intersecting at  $60^\circ$ .

**Density or Specific Gravity.**—The density or specific gravity of a body is the relation existing between its weight and the weight of an equal volume of water at a given temperature. In brief, the specific gravity of a substance is the weight of a cubic centimeter of its material in grams. Strictly considered, density is as much a chemical as a physical property of matter, for while its value depends partly upon the closeness with which a body's molecules are packed together, i.e., the number of molecules within a unit volume, it is affected also by the weights of these molecules—and the weights of the molecules depend upon the number, the nature, and the arrangement of their component atoms.

Since gravity acts upon a homogeneous mass of matter as a whole rather than upon its component parts independently, the result of its pull cannot be governed by the laws of symmetry. However, in the case of dimorphous substances (substances crystallizing in two different systems) the two forms will possess different specific weights. *Calcite* and *aragonite*, for instance, are both forms of  $\text{CaCO}_3$ , but the specific gravity of calcite, which is hexagonal, is 2.714 and that of aragonite, which is orthorhombic, is 2.94.

**Methods of Determining Densities.**—Theoretically, the simplest method of determining the specific gravity of a solid body is (1) to weigh a small fragment suspended from one arm of a balance by a silken thread, (2) to immerse in water and weigh again, (3) to subtract the weight in water from the weight in air, and (4) to divide this remainder into the original weight in air. The reasons for these manipulations are known to all students familiar with the principles of elementary physics.

If a fragment whose weight in air is 3.25 grams loses 1.25 grams when weighed in water, its density is  $3.25 \div 1.25$  or 2.6.

**The Jolly Balance.**—The most convenient instrument for the rapid determination of approximate densities is the Jolly

balance. This consists essentially of a spiral of wire fastened at the top to a movable arm (see Fig. 289). At the lower end it is provided with two little pans, one suspended beneath the other. The lower pan is kept always immersed to the same depth in water, while the other one hangs in the air. On the upright standard behind the spiral is a mirror on which is engraved or painted a scale of equal parts. The object whose density is to be determined is first weighed in the upper pan, then transferred to the lower pan and weighed again. The second weight subtracted from the first will give the weight of the water replaced by the fragment. The original weight divided by this yields the result desired.

The method of using the Jolly balance is simple. Before each weighing the zero point of the instrument is fixed by glancing along the end of the wire at its reflection in the mirror. The division of the scale that lies exactly in the line of sight between the point on the spiral and its image in the mirror is the zero point for that determination. A small fragment of the substance whose density is sought is then placed in the upper pan, care being taken to prevent the bottom of the pan from touching the water in which the lower pan is immersed—usually the vessel containing the water must be lowered immediately upon placing the fragment in the upper pan. When the instrument comes to rest and the vessel holding the water is adjusted until the surface of the water is as high above the lower pan as it was originally, a second reading is made. The fragment is then transferred to the lower pan, and the level of the water is again adjusted. A third reading is now taken. If the first reading

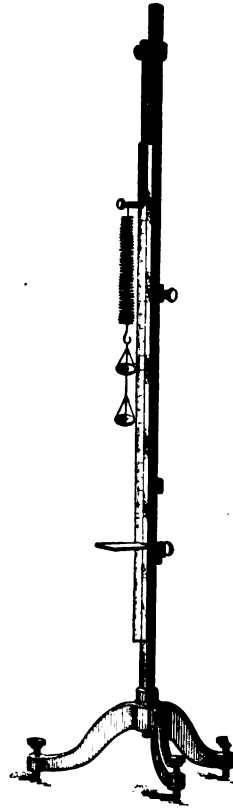


FIG. 289.—The jolly balance for approximate determinations of specific gravity.

be subtracted from the second and third readings the results will give the relative weights of the fragment in air and in water. The difference between these two results divided into the first weight will yield the specific gravity sought.

**The Use of Heavy Solutions.**—Often it is more convenient to determine the density of small fragments of crystals by some other method than either one of those described above. This is always true in the case of very small fragments and of powders.

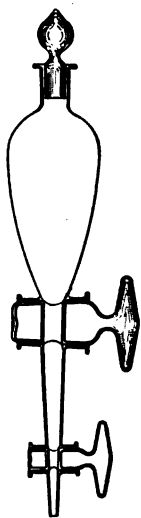


FIG. 290.—Separating funnel for use with solutions.

For the determination of the specific gravity of these use is made of solutions of high specific gravity, which by the addition of water or of some other liquid may have their densities lowered. The powder or small fragment is thrown into the solution. If it floats, the density of the solid is less than that of the liquid. This is gradually diluted with the proper medium until the fragments remain suspended in the mixed fluids, neither rising nor sinking. When this occurs fragments and liquid are of the same specific gravity. The density of the liquid may then be determined by any one of the physical methods appropriate for this purpose. Naturally the use of heavy solutions is limited to those substances whose densities are less than their own.

There are several solutions in common use for the determination of specific gravities. The one most frequently employed is a solution of mercuric and potassium iodides, known as the Thoulet solution. Its greatest density is 3.19. It will serve to determine the densities of substances lighter than 3.19, but, of course, cannot be used for heavier substances. Among the other heavy solutions often used for the same purpose may be mentioned the Klein solution (cadmium borotungstate in water) with a maximum density of 3.6, the Rohrbach solution (barium-mercury-iodide in water) with a density of 3.59, and the Retgers solution (thallium-mercuro-nitrate at 76°) with a specific gravity of 5.3.

**Practical Uses of Specific Gravity Determinations.—**

Fragments of substances that closely resemble each other in general appearance and in many of their physical and chemical properties may often be easily distinguished from one another by their densities, hence the determination of the specific gravity of a substance under investigation will often lead to correct inferences concerning its nature and identity. For example, *anorthite* and *albite* are two feldspars that resemble each other very closely in appearance. The former, however, has a specific gravity of 2.76 and the latter of 2.63.

Further, a mixture of substances in a finely divided form may be separated into its component parts by use of a heavy solution in a separating funnel, such as is represented in figure 290. The heaviest of the powders will fall and can be drawn off while the lighter ones remain floating on the solution.

**LIST OF SOME IMPORTANT MINERALS AND THEIR DENSITIES.**

Sulphur .....	2.05	Topaz .....	3.56
Rock salt.....	2.10	Garnet .....	3.75
Gypsum .....	2.30	Celestite .....	3.97
Orthoclase ....	2.56	Rutile .....	4.25
Quartz .....	2.65	Magnetite .....	5.20
Calcite .....	2.72	Hematite.....	5.30
Muscovite ....	2.85	Cassiterite.....	6.84
Biotite .....	3.01	Cinnabar .....	8.00

**METALS AND THEIR APPROXIMATE DENSITIES.**

Potassium ....	.86	Iron .....	7.80
Magnesium ...	1.80	Copper .....	8.90
Aluminium ....	2.50	Silver .....	10.60
Zinc .....	7.10	Gold .....	19.30

## CHAPTER XVIII.

### OPTICAL PROPERTIES OF CRYSTALS.

**Introduction.**—The light that falls upon a crystal is partly reflected from its surface, partly transmitted through its substance, and partly absorbed within it. In either case the light suffers changes which affect its character, and the nature of these changes is determined by the material of the crystal and by its physical symmetry.

**Properties Dependent upon Reflection.**—Whenever a ray of light falls upon a surface a portion of it is reflected in accordance with the simple law: *the angle of reflection is equal to the angle of incidence and both are in the same plane*. Moreover, some of reflected light suffers a change which causes it to possess certain properties not possessed by ordinary light. Because of the phenomena exhibited by this changed part of the reflected light, this part is described as being polarized; i.e., the vibrations that transmit it oscillate in a single plane parallel to the line of transmission.

The properties depending upon the character of the reflected light are color and luster.

**Color.**—The color of a crystallized substance, like that of other substances, depends upon the character of the light reflected from its surface, which in turn depends upon the color of the light that falls upon it, upon the color and quantity of that which is absorbed, and upon the quality of the reflecting surface.

When illuminated by light from different sources the same crystal often appears quite differently colored. The most remarkable case of this kind is the mineral *alexandrite*, or *chrysoberyl* ( $\text{BeAl}_2\text{O}_4$ ), the value of which as a gem depends upon the fact that it appears green by daylight and red by lamplight or gaslight. Indeed, the colors of substances vary so greatly in different lights that mineralogists have found it necessary to

agree that the colors mentioned in their descriptions of minerals shall refer only to the colors in daylight.

When white light falls upon any surface some portion of it is absorbed or transmitted, the remainder being reflected. If certain rays are absorbed or transmitted to a greater extent than certain others the reflected light will be of a different color from the incident light, for it will be made up of white light minus the rays that have been absorbed or transmitted. When white light falls upon a mass of *cinnabar* ( $\text{HgS}$ ), for instance, certain of its rays are absorbed. Those that remain and are reflected constitute red light, and the mineral appears red.

The color of opaque substances is much more characteristic than that of transparent or translucent ones. In the latter case the distinctive color (*ideochromatic*) is often obscured by impurities or by some pigment (*allochromatic*) which is present in such small quantity that its nature in many instances cannot be determined. Hence, not much reliance can be placed in the colors of such substances as distinctive characteristics. For instance, many minerals that are colorless when pure are found in nature with a great variety of tints. *Tourmaline* is a case in point. While transparent colorless crystals are known, the predominant colors are black, brown, green, and red.

Although opaque substances possess much more characteristic colors than do transparent or translucent ones, it frequently happens that their true colors are obscured by a surface tarnish. The true color is the color on a fresh fractured surface.

**Streak.**—Not only does the color of a substance vary with its nature, but it varies also with the character of its reflecting surfaces. More light is reflected from a smooth surface than from a rough one, and, conversely, more light is absorbed by a rough surface than by a smooth one. As a consequence, the light reflected from a smooth surface and that from a rough one may be made up of different components differently combined, and hence the colors of the two surfaces may appear different. A *pyrite* ( $\text{FeS}_2$ ) crystal has a bright, brassy-yellow color, while a rough-surfaced fragment of the same mineral may appear bronzy-yellow or greenish.

*A still greater contrast in color is often noted when crystals of a substance are compared with their powders.* This is especially true of opaque minerals and of translucent ones that are highly colored. While *pyrite* crystals are brassy-yellow, their powder is greenish-black. *Hematite* in crystals is often of a steel-black color. Its powder is blood-red or reddish-brown.

The color of the powder of a substance is more characteristic than that of its crystals, partly because in the powder the effect of variations in the quality of the reflecting surfaces is eliminated.

The most convenient method of obtaining a small quantity of crystal powder is by drawing a fragment of the crystal across a rough porcelain surface. *The mark left on the porcelain is known as the streak.* In all descriptions of mineral species the color of the streak is mentioned.

Transparent and translucent substances, whatever their color, usually possess a white streak. This is not so much due to differences in the absorbent power of large fragments and of their powder as it is to the dilution of the pigment that colors them. On the same principle a tumbler of water to which a little blue litmus solution is added has a bluish tint, while a single drop of the same water is practically colorless.

**Luster.**—The surfaces of crystals often present very decided peculiarities independently of color. Some surfaces reflect nearly all the light that falls upon them, others but a small quantity. The former glisten perceptibly, the latter are dull. Others reflect a portion of the light from their outer surfaces, and a portion from the surfaces of cracks, etc., within them. Some others scatter the light—diffuse it. The result of these different phenomena is known as *luster*.

The terms most frequently used in describing lusters are the following: *metallic*, *vitreous*, *adamantine*, *resinous*, *pearly*, and *silky*. The metallic luster is confined to opaque substances. It is the luster that is characteristic of the metals. The vitreous luster is that of glass; the adamantine, that of the diamond and of other very brilliant transparent substances; and the resinous, that of rosin. The pearly luster is found only in those substances that have a very perfect cleavage or that are traversed by numerous

cracks. The play of colors that is its characteristic feature is caused by the interference of a portion of the light reflected from the sides of the cleavage or other cracks, a phenomenon analogous to that of Newton's rings. It is so called because it is the luster of satin and of silk.

**The Transmission of Light.**—That portion of incident light which is not reflected from the surface of a substance penetrates its mass, and either passes through it or is stopped by it. That portion which passes through is said to be transmitted. The portion that is stopped is said to be absorbed. Nearly all substances transmit some light, though often to such a slight degree that they are practically opaque. An opaque substance transmits no light, therefore it appears dark when viewed from the side opposite to that on which the light is incident. A transparent substance transmits nearly all the light that penetrates its mass; i. e., nearly all the light that is not reflected from its surface. When viewed from the side opposite to that on which light is falling, it may be white or it may possess a distinct tint. An object can be seen distinctly through a transparent substance. A translucent substance transmits some light, but not enough to render distinctly visible an object viewed through it. A piece of iron is opaque, colorless glass is transparent, and porcelain translucent.

**Absorption of Light.**—The difference between opaque, transparent, and translucent substances is due to the difference in the quantity of light absorbed by them. The opaque bodies absorb all the light that is not reflected, the translucent ones absorb a considerable quantity of it, and the transparent bodies almost none.

In many translucent and transparent bodies light of a certain color may be absorbed while that of a different color is transmitted. As a result of this phenomenon the substance may have a distinct color in transmitted light which is usually different from that produced by reflected light. Thin sheets of gold are yellow by reflected light and green by transmitted light. The color of the light transmitted may be determined by the nature of the substance through which it passes or it may be determined by inclusions or by a pigment diffused through its mass. The former is



an *ideochromatic* color, and the latter *allochromatic*. Moreover, different portions of crystals may transmit light of different colors. This will happen when, during the crystal's growth, the solution from which it separated changed in composition from time to time, and consequently material of a slightly different character was successively added to the nucleus already in existence. The result is a distribution of color which is frequently zonal (see p. 150), as in *quartz*, *tourmaline*, *augite* or



FIG. 291.—Thin section of rock showing zonal structure in hornblende. Section cut parallel to the basal plane of the hornblende. Magnified 50 diameters. (After Rosenbusch).

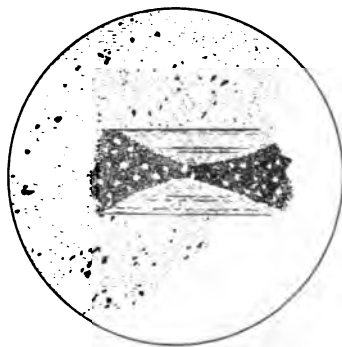


FIG. 292.—Vertical section of ottrelite in rock, showing regular distribution of color. Magnified 50 diameters. (After Rosenbusch).

*hornblende* (Fig. 291), and sometimes disposed in accordance with the presence of certain crystal planes, as in *ottrelite* or *augite* (Fig. 292).

Not only do different substances vary in their powers of absorption, but the same substance when in crystals may possess this power in different degrees along different directions. Light transmitted in a certain direction through a crystal may thus lose some of its rays, while other rays may be lost when transmission occurs through some other direction. Thus a crystal may appear differently colored when viewed by transmitted light in different directions.

**Pleochroism.**—This is the general term applied to the property of exhibiting different colors in different directions. If a crystal possesses different colors when viewed in two different

directions, it is said to be *dichroic*; if it exhibits three distinct colors when viewed in three different directions, it is *trichroic*.

In the case of amorphous and isometric substances the same colored light is absorbed irrespective of the direction of the rays transmitted through them. These bodies thus exhibit no pleochroism. In crystals possessing a principal plane of symmetry the absorption in the direction of the  $c$  axis is often different from that which takes place in the direction of the lateral axes. Hence *these crystals may be dichroic*.

A cube of dark *tourmaline*, an hexagonal mineral, when viewed through the direction of the  $c$  axis appears much darker than when viewed in the direction at right angles to this, and may possess different colors in the two directions. The absorption is greater for the ray transmitted parallel to  $c$  than for the rays transmitted parallel to the plane of the axes  $a$ . This fact is represented by the formula  $O > E$ . The symbol  $O$  refers to the ray transmitted in the direction of the  $c$  and  $E$  to the ray transmitted parallel to the principal plane of symmetry (see also page 205).

Only *crystals possessing no principal planes of symmetry may be trichroic*, the absorption being different along three directions perpendicular to one another; consequently, these crystals may appear of three different tints when viewed along these three different directions, and at the same time they may possess different degrees of transparency in these same directions.

The mineral *cordierite*, an orthorhombic mineral, is often strongly trichroic in light blue, dark blue and yellowish-white tints. *Glaucophane*, a monoclinic mineral, is blue, violet, and yellowish-green. *Axinite*, a triclinic mineral, is colorless, yellow, and violet. Nearly all transparent minerals are pleochroic in some degree, but frequently their differences in tint along different directions are so slight that they can be detected only by the aid of an instrument known as the *dichroscope*, which consists simply of a small rhomb of iceland spar (calcite) mounted in a brass tube closed at both ends except for two small holes which serve as peep-holes, and through which the crystal is viewed. The little rhomb causes double refraction (see pp. 204-205) and thus separates the two rays.

### Relation Between Pleochroism and Crystal Symmetry.—

Only crystallized substances can exhibit pleochroism, and not all these exhibit it. Opaque crystals can, of course, not exhibit it. Moreover, crystals that are completely transparent in every direction cannot exhibit it. These are colorless in all directions because there is no absorption. Isometric crystals likewise absorb equally in all directions and consequently must show a single color by transmitted light. Pleochroism is therefore limited to anisotropic substances—substances crystallizing in any system but the isometric—because only in anisotropic substances is light absorbed differently in different directions. *Hexagonal and tetragonal crystals may show dichroism, but they cannot exhibit trichroism. Orthorhombic, monoclinic, and triclinic crystals may be trichroic.\**

**Fluorescence and Phosphorescence.**—Some substances such as *fluorite* ( $\text{CaF}_2$ ) possess the *property of transmitting light of a certain color while at the same time radiating light of some other color*. This property is known as *fluorescence*. Fluorite may appear green by transmitted light, while at the same time it glows with a pale blue light. This property of fluorescing (from the name of the mineral best exhibiting it) is now known to be possessed by many crystals, such as those of the uranium minerals, and those containing fluorine and boron, and the manufactured substances fuchsin ( $\text{C}_{20}\text{H}_{19}\text{N}_3\text{HCl}$ ), fluorescein ( $\text{C}_{20}\text{H}_{12}\text{O}_8$ ), and magnesium cyanplatinite ( $\text{MgPt}(\text{CN})_4 + 7 \text{H}_2\text{O}$ ).

In the case of crystals of magnesium cyanplatinite and of some other substances it is noted that the fluorescence is different in color from different faces. In the cyanplatinite, which is tetragonal, the prismatic faces fluoresce green and the basal planes bluish-red. *In general, similar planes fluoresce alike, dissimilar planes may fluoresce differently.*

Other substances, for example, phosphorus, possess *the power of giving off light rays in the dark at comparatively low temper-*

\* The mineral dealers furnish at low prices little plates of mica, beryl, cordierite, tourmaline, and other minerals so mounted in cork cells that they may readily be revolved about an axis. On holding the plates opposite a window and revolving them, their differences in color along different directions are conveniently studied.

atures. Many specimens of *diamond* (C), *calcite* ( $\text{CaCO}_3$ ), etc., glow when removed from the presence of sunlight into a dark room. This property is known as phosphorescence. Other minerals, such as *fluorite* ( $\text{CaF}_2$ ) and *apatite* ( $\text{Ca}_5\text{P}_3\text{O}_{12}\text{Cl}$ ), glow brightly when heated to a temperature far below that of red heat. Others become luminous when their crystals are rubbed together, as, for instance, *sugar* and *zinc-blende* ( $\text{ZnS}$ ), and others glow when subjected to the action of the cathode rays or of radium emanations. The difference in the behavior of natural and imitation gems under the action of the cathode rays is sometimes made use of in detecting frauds.

**Refracted Light.**—If a ray of light traveling through air fall upon a transparent solid or liquid body obliquely, *it suffers a change in direction in its passage through the body; i.e., it is refracted.* Both the incident and the refracted rays are in the same plane, but the latter is bent toward the perpendicular to the surface of the body at the point of incidence of the former. For any given substance the amount of refraction or bending varies with the obliqueness of the incident ray, the refraction being greater as the angle of incidence becomes larger.

If in figure 293 CB represent the path of a ray of light falling on the surface of a substance at B, its angle of incidence is  $\angle CBA$  or  $i$ . This ray, upon entering the substance, suffers a change in direction, its new path being represented by BR. Its angle of refraction is  $r$ .

**The Index of Refraction.**—*For every uncrystallized substance and every substance crystallizing in the isometric system there is always a definite relation existing between the angle of incidence and the angle of refraction.* This relation is a constant one for light of a given color, provided the medium surrounding the body is the same in all cases. For example, the relation existing between the angle of incidence of a ray of light passing through air and

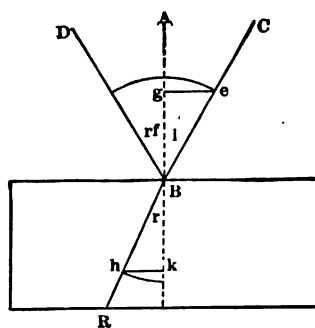


FIG. 293.

striking a surface of glass obliquely and its angle of refraction in the glass is always the same no matter what may be the size of the angle of incidence. The ratio is expressed by the formula  $n = \frac{\sin i}{\sin r}$ , in which  $n$  is the constant,  $i$  the angle of incidence, and  $r$  the angle of refraction (see Fig. 293).

The greater the amount of the bending, or refraction, of a ray, the smaller the angle  $r$ , hence the less the value  $\sin r$  as compared with the value  $\sin i$ , and the greater the constant  $n$ , or the greater the power of refraction of the body to which it refers.

The ratio  $n$ , as has already been stated, is a constant for all rays of light of the same color entering a homogeneous, amorphous, or isometric body from a given medium, as, for instance, air, irrespective of their obliquity. It differs, however, for different substances, and varies with the medium traversed by the rays before they enter the refracting substance, and also with the color of the light.

It is because of the differences in amount of refraction suffered by rays of different colors that the spectrum is produced when white light is allowed to traverse a glass prism.

When the constant  $n$  refers to the refraction of a ray of light of a definite color passing from a vacuum into a body, it is known as the *index of refraction*. It is characteristic for different substances, and is often employed for identifying them. The indices of refraction for several substances are as follows:

Water = 1.33. Air = 1.0003. Rock salt = 1.54. Diamond = 2.4195.  
Crown Glass = 1.53. Flint glass = 1.61.

**Single and Double Refraction.**—In all amorphous transparent bodies and in all those substances that crystallize in the isometric system the index of refraction remains the same, whatever the direction of the incident ray, provided the light is the same in color and the medium through which it passes is the same. *Such bodies are known as isotropic or singly refracting. In them there is only one refracted ray.*

Light falling upon substances that crystallize in any other system than the isometric is usually split up by refraction into two rays

following different paths (Fig. 294). These bodies are known as *anisotropic* or *doubly refracting*. Both of the refracted rays are polarized.

An excellent example of a doubly refracting substance is a cleavage piece of *calcite* or iceland spar ( $\text{CaCO}_3$ ), which on account of its strong doubly refracting power has long been called "double-spar." If a plate of the clear mineral be placed over a pin-hole in a piece of cardboard and both be held up to the light, the pin-hole will appear double (Fig. 294). If the plate be now revolved parallel to the plane of the cardboard one of the pin-



FIG. 294.—Photograph of double image of pin-hole as seen through a cleavage rhombohedron of calcite.

hole images will revolve about the other, which will remain fixed in position. *The light ray that produces the fixed image may easily be shown to obey the law of simple refraction—its index of refraction is a constant. This is called the ordinary ray. The ray that produces the other image obeys another law. It is called the extraordinary ray. Its index of refraction varies with the direction pursued by the transmitted ray. In calcite the index of refraction of the ordinary ray (written  $\omega$ ) is 1.6585 for yellow light, while that of the extraordinary ray varies between 1.6585 and 1.4863. The value that differs most from  $\omega$  is taken as the*

index of refraction for the extraordinary ray (written  $\epsilon$ ). In this instance  $\omega > \epsilon$ . The mineral is said to be *optically negative*. In quartz, on the other hand,  $\omega$  for yellow light is 1.5442 and  $\epsilon$  is 1.5533; the mineral is *positive*.

The explanation of the double image seen in calcite is readily understood by reference to figure 295. The ray BC striking the calcite plate at C is transmitted through it along two paths, CD and CE. These emerge as two rays, DF and EG. The eye receiving the light at F sees the ray as though originating at I, while the light received along the line EG appears to originate at J. Hence, two images of the source of light are seen—one at I

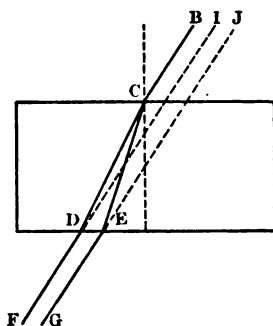


FIG. 295.

and the other at J. If a black spot be at B, likewise two images of the spot will be seen at I and J.

**Uniaxial and Biaxial Crystals.**—In some anisotropic bodies, like quartz and calcite, one of the two rays into which the refracted light is divided possesses a constant index of refraction, while the other possesses a variable index. In other anisotropic bodies both refracted rays possess variable indices. Neither can be spoken of as the ordinary ray; both are extraordinary. The first class of bodies embraces substances that crystallize in the hexagonal and tetragonal systems; i.e., in systems possessing a principal crystallographic axis (the vertical axis) differing from a series of equivalent lateral axes. The second class embraces orthorhombic, monoclinic and triclinic crystals, or those with three unequivalent crystallographic axes.

In bodies of the first class there is one direction; viz., parallel to the vertical axis, along which a ray is transmitted without double refraction. This is the ordinary ray. In bodies of the second class there are two directions along which no double refraction takes place, and these two directions are inclined to one another at angles varying with the body and with the color of the light.

**Optical Axes and the Axial Angles.**—The directions in anisotropic bodies along which no double refraction takes place are known as optical axes. Crystals of the first class, as defined above, possess but one of these axes, hence they are said to be *uniaxial*. Those of the second class possess two optical axes, and are consequently said to be *biaxial*.

The angle of inclination between the optical axes of biaxial

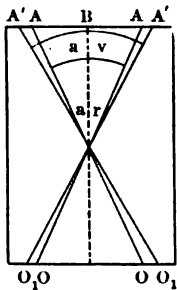


FIG. 296.

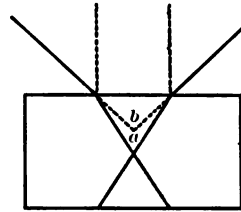


FIG. 297.

crystals is known as the *optical axial angle*. It can very readily be measured. When once known, it serves as an important means for distinguishing between two substances resembling each other in other respects.

In figure 296 is represented a section through a biaxial crystal in the plane of the optical axes. The light is supposed to pass into the lower side of the crystal as a bundle of rays converging within the crystal. The lines AO represent the directions along which there is no double refraction of red rays, and A'O', the directions along which there is no double refraction of violet rays. The optical angle for the red ray is measured by the arc between A and A and that for violet light by the arc between A' and A'.



The value of the optical angle as given in the text-books on crystallography is usually the *true axial angle*; i.e., the *axial angle within the crystal*. This may be determined from the apparent axial angle, the value of which is obtained by observation in air. The apparent angle is always greater than the true angle (Fig. 297), for the rays of light, passing from the denser crystal into the rarer medium surrounding it, are bent away from the perpendicular, and so are inclined to each other at a greater angle than while traversing the denser crystal. In the figure, *b* is the apparent optical axial angle and *a* the true angle. In order to reduce the size of the apparent axial angle so that it may be seen more easily the observation is frequently made upon a crystal plate immersed in water, oil, or in some highly refracting liquid. From the data obtained for this observation the true axial angle is calculated.

The apparent axial angles of *hypersthene*, *bronzië*, and *enstatite* when measured in oil are  $85^{\circ} 39'$ ,  $112^{\circ} 30'$ , and  $133^{\circ} 8'$ . For *olivine* ( $\text{Mg}_2\text{SiO}_4$ ) the apparent and true axial angles for differently colored rays are:

	Red	Yellow	Blue
Apparent angle in oil.....	$100^{\circ} 52'$	$101^{\circ} 2'$	$101^{\circ} 30'$
True angle.....	$86^{\circ} 1'$	$86^{\circ} 10'$	$86^{\circ} 32'$

NOTE.—The relations of crystallized minerals toward light rays transmitted through them are too complicated to be discussed at greater length in an elementary text-book of this kind. Suffice it to state that the simple principles enunciated above have been made the basis of much study, and that the discussions to which this study has given rise are treated in large volumes devoted exclusively to the optical properties of minerals. By means of these properties the physical symmetry of crystals is most beautifully shown. To one who intends making a special study of crystals or a study of rocks, a knowledge of the optical properties of crystallized substances is indispensable.

## CHAPTER XIX.

### THERMAL PROPERTIES OF CRYSTALS.

**Transmission of Heat.**—Since heat and light are so similar physically, it is not to be wondered at that crystals conduct themselves toward heat rays very much as they do toward rays of light. Just as there are substances that are opaque and others that are transparent to light, there are those that are opaque and those that are transparent to heat. Substances that are transparent to light are not necessarily transparent to heat, and *vice versa*. *Sylvite* ( $\text{KCl}$ ), *rock salt* ( $\text{NaCl}$ ), and *alum* ( $\text{KAl}(\text{SO}_4)_2 + 12 \text{H}_2\text{O}$ ) are all transparent to light. Only the first two, however, will allow heat to pass through them. Alum is practically opaque to it. *Substances that are transparent to heat are said to be diathermous.*

Alum is opaque to heat when in solution, as well as when in crystals. Tanks of alum solution are often placed between the source of the light in projecting lanterns and the object the image of which is desired, in order to cut off the heat and protect the object from injury.

Most of the laws that are known to govern the action of crystals toward transmitted light are found to be applicable also in the case of heat. For instance, there are crystals that are singly refracting and others that are doubly refracting toward heat rays, and of the doubly refracting crystals some are uniaxial and others biaxial.

There are a few results of the action of heat upon crystals, however, that find no analogies in the action of light upon them; by conduction crystals (together with all other substances) become heated to their most distant parts; by the addition of a moderate amount of heat they expand or contract, and by the addition of a greater quantity they melt or become vaporized.

**Conduction of Heat.**—Substances differ in their capacity

to conduct heat. Of the metals, silver is the best conductor and bismuth the worst. Non-metallic substances are poorer conductors than is bismuth. They vary greatly in conductive capacity, just as do the metals. Moreover, *crystallized substances conduct with different rates in different directions.*

The relative conductive capacities of a crystal in different directions may be investigated by covering its faces with a thin film of wax and touching the wax with a needle point kept hot by an electric current. The wax will melt around the hot point for distances varying with the rapidity with which the heat is conducted in different directions. By inspection of the melted area the relative conductive capacity in these different directions may easily be determined. The lines drawn around the melted area at different stages in its production are called isothermal lines. They constitute curves analogous to the curves of hardness.

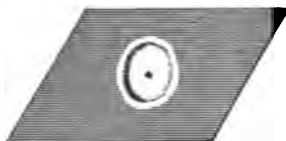


FIG. 298.—Diagrammatic illustration of variation in rate of heat conductivity in biaxial crystals.

The isothermal lines on the surfaces of amorphous bodies and on the faces of isometric crystals are always circles, indicating that in these bodies the conductive capacity does not vary with direction. In the case of hexagonal and tetragonal crystals (uniaxial crystals), the isotherms on basal planes are circles, but those on the other faces are ellipses. These curves indicate that while conduction is equally rapid in directions at right angles to the vertical axis, in other directions its rate is either greater or less than this, and in the direction of the vertical axis it is a maximum or minimum. The isotherms on the faces of crystals in the remaining systems show that the conductivity varies, being greatest and least along directions perpendicular to one another and intermediate along intermediate directions (see Fig. 298).

**Expansion and Contraction.**—Solids and liquids, as a rule, expand when heated and contract when cooled, the change in volume varying with the substance.

The ratio between the length ( $l$ ) of a bar of a given substance at  $0^\circ$  and its increase in length when heated to  $100^\circ$  ( $l'$  = length

at  $100^{\circ}$ ) is known as its coefficient of expansion. *This is a constant for each substance when measured under similar conditions.*

Exceptions to the above rule exist, but they are confined to substances when near their fusing points; i.e., when they are about to change their state from liquid to solid or vice versa.

When subjected to a varying temperature, *amorphous substances and isometric crystals expand and contract equally in different directions*; i.e., they possess but a single coefficient of expansion, while *uniaxial crystals expand and contract more or less in the direction of their vertical axis than in any other direction, and equally in directions perpendicular to this axis*; i.e., they possess a maximum and a minimum coefficient of expansion. *Orthorhombic, monoclinic and triclinic crystals (biaxial crystals) expand and contract differently in different directions.*

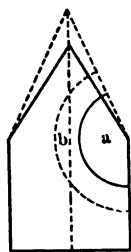


FIG. 299.

Although not discernible to the eye, the differences in length caused by varying temperature in the case of the axes of all crystals but those belonging to the isometric system are sufficiently great to affect the values of their interfacial angles and, consequently, their axial ratios (see Fig. 299). Hence, in the determination of axial ratios, it is often necessary to note the temperature at which the measurement serving as the basis of calculation was made in order that the ratio may have a definite meaning.

Crystals of *quartz* (hexagonal  $\text{SiO}_2$ ) increase by .000781 of their lengths and .001419 of their breadths when their temperature is increased  $100^{\circ}\text{C}$ . Their *a* axes increase in length about twice as much as their *c* axes, and thus their axial ratio diminishes with an increase in temperature.

**Fusion and Vaporization.**—When the temperature of a body is increased sufficiently, if it is not decomposed, it becomes vaporized, usually passing through the liquid state before becoming a gas. The temperature at which a body changes from a liquid to a solid, or vice versa, is known as its fusing point; the temperature at which it changes to a gas is its boiling point. The fusing and boiling points differ for different substances, but are

constant for the same substances under similar conditions. Since they are determined by the nature of the molecules of which the body is composed, and not upon their arrangement, they cannot be governed by the laws of symmetry.

TABLE OF FUSING POINTS.

*Metals.*

Tin.....	228°	Silver .....	960°
Cadmium .....	320°	Gold .....	1062°
Lead .....	335°	Copper .....	1083°
Zinc .....	419°	Nickel .....	1452°
Antimony .....	630°	Cobalt .....	1490°
Aluminium ....	658°	Platinum ....	1755°

*Non-metallic Substances.*

Ice .....	0°	Albite (NaAlSi <sub>3</sub> O <sub>8</sub> ) ...	1230°
Sulphur.....	114.5°	Fluorite (CaF <sub>2</sub> ) .....	1387°
Nitre .....	345°	Anorthite (CaAl <sub>2</sub> (SiO <sub>4</sub> ) <sub>2</sub> )	1532°

**Scale of Fusibility.**—The exact determination of the fusing point of a crystallized body often serves as an important aid in identifying it. The methods used for this purpose are, however, either very complicated, as in the case of substances with high melting points, or, when simple, they are especially adapted to the determination of low fusing points, such as those of many organic compounds. The latter are described in manuals on organic chemistry, and the former in treatises on heat or in special articles on the measurement of high temperatures.

Although it is not always practicable to determine the accurate melting point of a substance, it is frequently important to obtain at least some notion of its relative fusibility as compared with other substances. For this purpose the mineralogist von Kobell has proposed the following seven minerals arranged in the order of their fusibilities, to serve as a scale with which to compare other substances.

## VON KOBELL'S SCALE OF FUSIBILITY.

1. Stibnite. Melts in the flame of a candle.
2. Natrolite. Readily melts to a globule in a blowpipe flame.
3. Almandine. Difficultly fusible to a globule in the blowpipe flame.
4. Asbestus. The ends of fine splinters fuse to globules before the blowpipe.
5. Orthoclase. The sharp corners of small fragments may be rounded before the blowpipe.
6. Bronzite. The sharp corners of small fragments show the merest trace of rounding in the flame of powerful blowpipe.
7. Quartz. Infusible.

Minerals are said to fuse at 1, 2, 3, etc., when they melt as easily as stibnite, natrolite, almandine, etc.

Very small fragments should always be used in comparing fusibilities. They may be held in the loop of a piece of platinum wire fused into the end of a glass tube.

## CHAPTER XX.

### ELECTRICAL AND MAGNETIC PROPERTIES OF CRYSTALS.

**Magnetism of Bodies.**—All substances are believed to be magnetic in some degree, although in most of them the magnetic power is so slight as to be overlooked.

A few metals like iron and compounds of metals like magnetite ( $\text{Fe}_3\text{O}_4$ ) are strongly magnetic, often possessing poles like an ordinary magnet. The majority of compounds, however, are not endowed with the property of attracting other bodies, though they may themselves be attracted by or repelled from a strong magnet. *Those substances that are attracted by both poles of an ordinary magnet are said to be paramagnetic, while those that are repelled from both poles are diamagnetic.* When rods of a paramagnetic substance are suspended between the poles of a U-shaped electromagnet, they place themselves with their long axes in the line joining its poles. If the rods be of diamagnetic substances, their long axes take positions at right angles to this line.

The force with which anisotropic crystals are attracted or repelled by a magnet varies with the direction along which it acts. The directions along which greatest and least forces are exerted accord with the directions of the axes of morphological symmetry just as do the directions of heat propagation, etc.

*Amorphous bodies and isometric crystals are similarly paramagnetic or diamagnetic in all directions. In tetragonal and hexagonal crystals the magnetic force exerts a maximum and a minimum power in directions parallel with, and perpendicular to, the vertical axis, and in all directions perpendicular to the axis it is equal. In crystals of a lower grade of symmetry the force varies in three rectangular directions.*

**Electrical Properties.**—Crystals may become charged with electricity by one or another of many ways. Many may be made

electrical by rubbing, others by compression, others by fracturing, others by heating, etc. Only those crystals that are bad conductors manifest electrical properties to any great degree, since in the case of good conductors the electricity flows away as fast as it is generated unless the crystals be insulated. Crystallized substances may therefore be separated into two groups: (a) conductors, which may be made electrical but which do not retain their electricity, and (b) non-conductors, which, when they become electrified, maintain their electrical condition for some time.

**Conduction of Electricity.**—Crystals that are good electrical conductors behave toward electricity in a manner analogous to the behavior they exhibit with respect to heat. *Amorphous and isometric minerals conduct with equal facility in different directions. Those with one principal plane of symmetry conduct most rapidly along a direction parallel to the c axis, and least rapidly along directions perpendicular thereto, or vice versa. The optically biaxial minerals conduct with different degrees of facility along different directions.*

**Thermoelectrical Properties.**—When two strips of different metals are brought into contact at both ends and one end of the "couple" is heated, an electrical current is generated. The metal from which the current flows at the cold end of the couple is positively thermoelectric, and the other negatively thermoelectric. In general, when any two conductors are so united and subjected to differences in temperature at their different junctions, a current of electricity will flow from one to the other.

Many crystallized minerals that are good conductors exhibit thermoelectrical properties to a marked degree. When two crystals of *pyrite* are brought into contact and heated, a very strong current is set up between them, provided the crystals are not morphologically identical. Pyritoids  $\left( \begin{smallmatrix} \infty & Om \\ & 2 \end{smallmatrix} \right)$  of pyrite whose faces are striated parallel to the cubic faces are thermoelectrically positive (Fig. 300, while those striated perpendicularly to these faces are negative (Fig. 301). The current thus generated is stronger than the strongest current generated in a similar manner between any two metals.



The force of the current generated between two different crystal individuals varies with the nature of these individuals and with their morphological features. The best-known producers of thermoelectric currents are hemihedral and tetartohedral crystals.

Thermoelectrical currents are, however, not limited to "couples" of individual crystals. Rods cut from a single crystal will often generate a thermoelectrical current when they are so placed in contact that the same crystallographic directions in

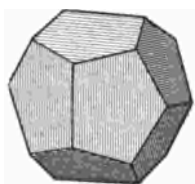


FIG. 300.

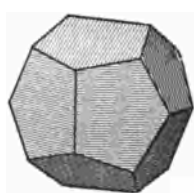


FIG. 301.

Two crystals of pyrite illustrating relation between striations and thermoelectrical properties of this mineral. Fig. 300 represents a positive, and Fig. 301 a negative crystal.

them are not parallel to one another. A law of direction thus applies to the exhibition of the thermoelectrical as well as to that of the other properties of crystals.

**Pyroelectricity and Pyroelectrical Properties.**—Non-conductors of electricity when crystallized exhibit certain peculiar electrical phenomena different from those exhibited by conductors.

In many instances when the temperature of a crystal or of a fragment of a crystal is rapidly changing it becomes charged with electricity, one end becoming positively electrified and other end negatively charged. That end charged with positive electricity during the rising of the temperature and with negative electricity when the temperature is falling is called the *analogue pole*. The end which is negatively charged with rising temperature and positively charged with falling temperature is the *antilogue pole*. The electricity produced in a body by changes in temperature is known as pyroelectricity.

*The distribution of pyroelectricity is governed entirely by the*

*symmetry of the crystals that exhibit it. Hemimorphic crystals have the two ends of their unsymmetrical axis differently charged, as have also all other crystals possessing a single polar axis of symmetry, such as certain hemihedral and tetartohedral forms. In these cases fragments of the crystals exhibit the same pyroelectric properties as complete crystals.*

*In crystals with several polar axes of symmetry the electricity is distributed over faces and solid angles in such a manner as to bring out clearly the symmetry of the form—morphologically similar*

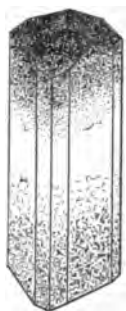


FIG. 302.—Tourmaline crystal after powdering with a mixture of sulphur and minium. Its upper end is the analogue pole.



FIG. 303.—Boracite crystal dusted with sulphur and minium to show distribution of pyroelectric properties. The darker dotting represents the minium.

*parts always being similarly electrified. In general, the opposite ends of symmetry axes become similarly electrified when they are equivalent morphologically, and differently electrified when they are morphologically unequivalent.*

Amorphous substances and holohedral regular crystals show no pyroelectrical phenomena.

The pyroelectrical properties of crystals may be conveniently investigated with the aid of very simple apparatus. The crystal to be studied is placed in an air-bath and brought to a fairly high temperature. It is then taken from the air-bath, quickly passed through the flame of a spirit lamp, and then laid on a piece of paper in a cold room. After it has lain undisturbed for a few moments it is dusted through a little sieve with a mixture of

pulverized sulphur and minium. The sulphur in passing through the meshes of the sieve becomes negatively electrified and the minium positively charged. The former is attracted to the positive poles of the crystal which are thus colored yellow, and the minium to the negative poles which are colored red. The distribution of the two colors and their intensity correspond to the distribution and intensity of the different charges of electricity on the crystal.

Figures 302 and 303 represent the appearance of a cooling *tourmaline* crystal and of a cooling *boracite* crystal after dusting with sulphur and minium. The *tourmaline* is hemimorphic. Its upper end is negatively electrified—it is the analogue pole—and its lower end positively electrified. This is the antilogue pole. The *boracite* crystal is apparently a combination of inclined hemihedral forms of the isometric system. The symmetry of its pyroelectric properties is in accord with the morphological symmetry of such a combination.

## CHAPTER XXI.

### SOLUTION AND ETCHED FIGURES.

When a substance is subjected to the action of solvents its corners will usually be dissolved more rapidly than the plane or curved surfaces upon it, and no definite laws of solution will be observed unless the substance be a crystal or a portion of a crystal. The rapidity with which the different parts dissolve is governed by the conditions under which the body exists with respect to the solvent. In the case of crystallized bodies, however, the rapidity of solution is governed largely by the laws of symmetry. In every crystal there are directions along which solution occurs at different rates. *Under similar conditions the crystals of a given body will always dissolve more rapidly along a certain direction than along any other direction.* Under different conditions, however, the maximum and minimum directions may be different. Indeed, a maximum direction for one solvent may become a direction of minimum solution for some other solvent. A similar condition is true with respect also to decomposing agents. A crystal will often suffer decomposition at different rates along different directions.

A crystal of the isometric mineral *fluorite* ( $\text{CaF}_2$ ) when subjected for a minute to the action of a dilute solution of hydrochloric acid (one part of a 36 per cent. solution of  $\text{HCl}$  in 1 part of  $\text{H}_2\text{O}$ ) loses a layer .006 mm. thick from the  $\infty \text{O} \infty$  faces, .01 mm. from the O faces, and a little more from the  $\infty \text{O}$  faces. When treated with a concentrated solution of soda for the same length of time the  $\infty \text{O} \infty$  faces lose a layer of material .016 mm. in thickness, while the O faces lose only .006 mm. and the  $\infty \text{O}$  faces .007 mm.

At the same time that the solvent is dissolving the exposed portions of a crystal it may eat its way into its interior along planes that may or may not be identical with the cleavage planes. *These*

planes are known as *solution planes*. Their positions are determined by the nature of the solvent, that of the substance of the crystal and its symmetry. Often the products formed by the solution are precipitated in the places where formed, or the cavities produced by their removal are subsequently filled with some new substance and thus inclusions result. Inclusions of this kind often indicate their mode of origin by their arrangement in definite planes.



FIG. 304.—Inclusions produced along solution planes in augite by the alteration of its substance. Magnified.

The accompanying figure (Fig. 304) represents the appearance of a thin slice of an altered *augite* when viewed under the microscope. The black areas represent purplish-black inclusions in the almost colorless augite. Note their arrangement in distinct lines.

Observations on the rapidity with which solution takes place may sometimes serve as a means of detecting the mode of crystallization of a substance, when crystal faces are not present upon it, or they may serve to distinguish between holohehedral, hemihedral, and tetartohedral forms, or between the various kinds of hemihedrism or tetartohedrism; i.e., they may disclose its grade of symmetry.

Professors Meyer and Penfield found that spheres cut from right-handed and left-handed tetartohedral crystals of quartz can easily be distinguished from one another by subjecting them to the solvent action of strong hydrofluoric acid.

**Etched Figures.**—A close inspection of the faces of a crystal that has been treated with a solvent will reveal the presence on



FIG. 305.—Result of treating with hydrofluoric acid a sphere of quartz cut from right-hand crystal. Viewed along the vertical axis. There was practically no solution at the positive ends of the lateral axes. (After Penfield.)

them of numerous little hollows and prominences whose shapes vary with the nature of the substance, the nature of the solvent, the temperature at which it acts, and the symmetry of the face acted upon. These are known as etch or etched figures. Their shapes accord so well with the symmetry of the faces acted upon that they have frequently been used to determine the symmetry class to which the crystals belong. For instance, by the observation of such figures produced on cubic crystals of various substances it is possible to distinguish between holohedral, hemi-

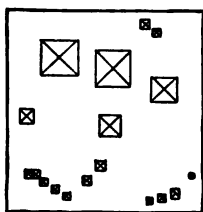


FIG. 306.

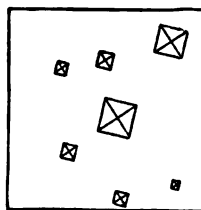


FIG. 307.

Etch figures on cubic faces of halite (Fig. 306) and sylvite (Fig. 307), showing that the two cubes possess different grades of symmetry.

hedral, and tetartohedral cubes. The etch figures on cubes of *halite* (NaCl), for example, exhibit holohedral symmetry (Fig. 306), while those on cubes of *sylvite* (KCl) indicate the symmetry of gyroidal hemihedrism (see Fig. 307).

All the etch figures are bounded by planes that are subject to the same crystallographic laws as the planes on the crystals of the etched substance. By their means it has been learned that many crystals that were formerly supposed to be holohedral are in reality hemihedral, being made up of combinations of plus and minus, or of right and left hemihedrons. In this way cubes of sylvite, as indicated above, have been shown to be gyroidally hemihedral (Fig. 307), hexagonal pyramids of *apatite* have been shown to be pyramidal-hemihedral, and the pyramid face of *quartz* to be tetartohedral.

*Muscovite* for a long time was believed to be orthorhombic. It possesses, however, the optical properties of a monoclinic mineral. Etching with hydrofluoric acid produces little figures

on the basal plane somewhat similar in shape to those shown in figure 308. These are symmetrical about a single plane. Muscovite thus is in reality a monoclinic mineral with an orthorhombic habit. The figures produced on the macropinacoid of *calamine*

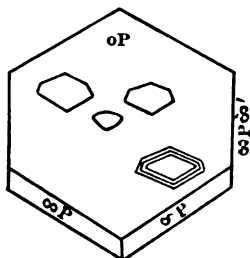


FIG. 308.—Etch figures on basal plane of muscovite which prove the mineral to be monoclinic.

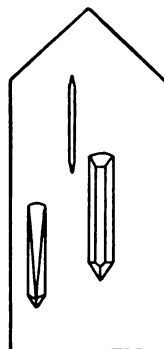
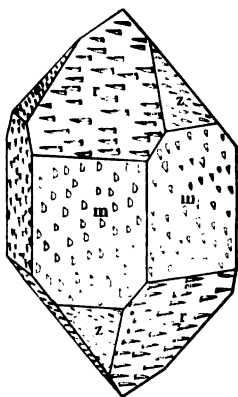
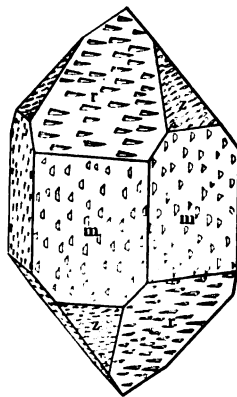


FIG. 309.—Etch figures on orthopinacoid of calamine showing its hemimorphic character.



A



B

FIG. 310.—Etch figures on right-hand (A) and left-hand (B) quartz crystals, disclosing their low grade of symmetry. (After Penfield.)

(orthorhombic  $\text{H}_2\text{Zn}_2\text{SiO}_5$ ) by dilute hydrochloric acid are indicated in figure 309. They are symmetrical about a vertical line, but are terminated differently. The mineral is hemimorphic. The pyramidal faces of quartz appear to belong to

the hexagonal pyramid. Etching with hydrofluoric acid produces unsymmetrical figures that are differently arranged on contiguous planes, but similarly arranged on alternate ones (see Fig. 310). The pyramid thus breaks up into two rhombohedrons, and both of these are unsymmetrical. The forms are tetartohedral. Figure 311 is a reproduction of a micro-photograph of the



FIG. 311.—Photograph of greatly magnified etch figures on the basal plane of apatite. (*After Baumhauer.*)

etched figures produced on the basal plane of apatite ( $\text{Ca}_3(\text{PO}_4)_2 + (\text{CaCl}) \cdot \text{CaPO}_4$ ) by dilute sulphuric acid. Usually the figures are not as sharply defined as these, but they are nearly always sufficiently distinct to exhibit clearly the symmetry of hexagonal pyramids of the third order.

The amount of work done in the investigation of etched figures has been enormous, and the results reached as the result of this study have had an immense influence upon the views of mineralogists concerning crystallization.





**PART III.**  
**CHEMICAL CRYSTALLOGRAPHY.**



## CHAPTER XXII.

### ISOMORPHISM AND POLYMORPHISM.

**Chemical Compounds.**—Crystallized substances are definite chemical compounds the characteristics of which are dependent upon the nature, the number, and the arrangement of the chemical atoms in their molecules, and upon the arrangement of their molecules. We usually refer to the nature and the number of the atoms in the molecule as its composition, while we designate the arrangement of the atoms as its constitution. Two substances may possess the same composition, but be differently constituted, hence may be different things, endowed with different properties. Differences in the arrangement of the molecules produce differences in crystallization.

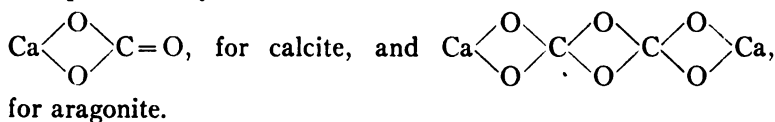
The difference in crystallization and in the physical properties of two substances that are identical in composition, as, for instance, the hexagonal and the orthorhombic forms of  $\text{CaCO}_3$ , may be due to differences in the constitution of their molecules, and this difference in constitution may be the ultimate cause of their difference in crystallization. Since the two substances are different things exhibiting different properties, they have been given different names. One, the hexagonal form, is called *calcite*, while the other has been named *aragonite*.

**Polymorphism.**—When a substance crystallizes in two distinct forms it is said to be *dimorphous*; when in three forms, *trimorphous*, etc. In general it is said to be *polymorphous*.

We can scarcely speak with accuracy of one substance crystallizing in several forms. If, as has been assumed, the morphological properties of bodies are functions of their chemical composition, it must necessarily follow that bodies with different properties are different substances chemically. Two substances possessing exactly the same number of the same elements in their molecules may, nevertheless, be distinct substances by virtue of the difference in the arrangement of these elements within the molecules. They are frequently spoken of as the same substance,

because their composition may be represented by the same chemical formula (*cf.* p. 4).

For instance, the carbonate of calcium, which may be represented by the formula  $\text{CaCO}_3$ , crystallizes in the hexagonal system as calcite, and in the orthorhombic system as aragonite, as has already been stated. It is therefore said to be dimorphous. It is probable, however, that one of these substances would better be represented by the formula  $(\text{CaCO}_3)_x$ , signifying that its molecule contains more atoms than does the molecule of the other. If  $x=2$ , the formula becomes  $(\text{CaCO}_3)_2$ , in which case the difference in composition between the two substances might be represented by the constitutional formulas:



Polymorphous modifications of a substance are the result of variations in the conditions under which its different forms are produced. These differ in purely physical properties as widely as they do in geometrical properties. Orthorhombic sulphur crystallizes from solution. Monoclinic sulphur is obtained by cooling a fused mass. The one is transformed into the other at about  $96^\circ$ . Orthorhombic sulphur melts at  $113.5^\circ$ , and monoclinic sulphur at  $119.5^\circ$ . The density of the former is 2.05 and of the latter 1.96.

Mercuric iodide is another substance possessing well-known modifications. From solution it crystallizes as red tetragonal crystals; but from a fused mass it separates as yellow orthorhombic crystals. The red variety passes into the yellow variety when it is heated to  $126^\circ$ .

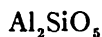
#### PARTIAL LIST OF POLYMORPHOUS BODIES.

##### *Dimorphs.*

Pyrite (regular)	$\text{FeS}_2$	(orthorhombic) Marcasite
Arsenolite (regular)	$\text{As}_2\text{O}_3$	(monoclinic) Claudetite
Calcite (hexagonal)	$\text{CaCO}_3$	(orthorhombic) Aragonite
Yellow (orthorhombic)	$\text{HgI}_2$	(tetragonal) red

*Trimorphs*

Anatase (tetragonal, $a : c = 1 : .71$ )	Sp.Gr. = 3.84. Double Refraction —.
Rutile (tetragonal, $a : c = 1 : .64$ )	Sp.Gr. = 4.24. Double Refraction +.
Brookite (orthorhombic)	Sp.Gr. = 4.06. Double Refraction +.



Cyanite (triclinic)	Sp.Gr. = 3.60. Hardness 5-7.
Sillimanite (orthorhombic)	Sp.Gr. = 3.24. Hardness 6-7.
Andalusite (orthorhombic)	Sp.Gr. = 3.18. Hardness 7.



Quartz (hexag.)	Tridymite (orthorh.)	Asmanite (tetrag.)
-----------------	----------------------	--------------------

*Tetramorph.*

Sulphur may be orthorhombic, monoclinic with  $a : b : c = .996 : 1 : .999$ ,  $\beta = 95^\circ 46'$ , monoclinic with  $a : b : c = 1.06 : 1 : .71$ ,  $\beta = 91^\circ 47'$ , and hexagonal.

**Isomorphous Compounds.**—A comparative study of crystallized bodies has shown that those possessing analogous compositions often possess also the same general crystalline form.

Crystals are of the same general crystalline form when they possess the same grade of symmetry and the same habit and have their corresponding interfacial angles of nearly the same value. Compounds possess analogous compositions when their formulas are of the same type, as  $\text{Na}_2\text{CO}_3$  and  $\text{K}_2\text{CO}_3$ , or  $\text{Na}_3\text{PO}_4$ ,  $\text{Na}_3\text{AsO}_4$  and  $\text{K}_3\text{AsO}_4$ .

Analogous compounds may be regarded as derived from one another or from some common source by the replacement of single elements or groups of elements by certain other nearly allied elements or groups of elements. For instance, by replacement of the hydrogen in  $\text{H}_2\text{CO}_3$  the following series of compounds may be derived:  $\text{MgCO}_3$ ,  $\text{CaCO}_3$ ,  $\text{FeCO}_3$ ,  $\text{MnCO}_3$ , etc. These possess analogous compositions.

*Isomorphous bodies*, in brief, are those possessing similar crystal forms. Isomorphism, or the property of being isomorphous, is limited to bodies of analogous compositions.

The following short list of isomorphous minerals will illustrate the meaning of the term:

#### PARTIAL LIST OF ISOMORPHOUS GROUPS.

<i>Orthorhombic</i>		<i>Hexagonal</i>		<i>Orthorhombic</i>	
Forsterite	$\text{Mg}_2\text{SiO}_4$	Calcite	$\text{CaCO}_3$	Diaspore	$\text{AlO}(\text{OH})$
Tephroite	$\text{Mn}_2\text{SiO}_4$	Magnesite	$\text{MgCO}_3$	Goethite	$\text{FeO}(\text{OH})$
Fayalite	$\text{Fe}_2\text{SiO}_4$	Sphercobaltite	$\text{CoCO}_3$	Manganite	$\text{MnO}(\text{OH})$
		Siderite	$\text{FeCO}_3$		
		Rhodochrosite	$\text{MnCO}_3$		
		Smithsonite	$\text{ZnCO}_3$		

**Morphotropism.**—Although isomorphous compounds possess the same general crystallographic habit, there are no two of them exactly alike. With a change in the chemical composition of any substance there is a corresponding, though sometimes but slight, change in its morphological and physical properties. *That partial change which is effected in the crystallization of a substance by the replacement of one of its constituents by some other element or group of elements is known as morphotropism.* This usually consists in a slight change in the axial ratio of its crystals and a corresponding change in the values of their interfacial angles. The change in morphological properties is, of course, attended with changes in physical properties. The character and amount of morphotropic change produced by the introduction of an element, or group of elements, into any compound is known as its morphotropic action.

The variation in axial ratio shown by members of two isomorphous groups is illustrated below:

<i>Hexagonal-Hemihedral</i>		<i>Orthorhombic</i>	
Calcite, $\text{CaCO}_3$ ,	$a : c = 1 : 0.8543$	$\text{Mg}_2\text{SiO}_4$ ,	$a : b : c = .4666 : 1 : .5868$
Rhodochrosite, $\text{MnCO}_3$ ,	$a : c = 1 : 0.8259$	$\text{Mn}_2\text{SiO}_4$ ,	$a : b : c = .4621 : 1 : .5914$
Siderite, $\text{FeCO}_3$ ,	$a : c = 1 : 0.8191$	$\text{Fe}_2\text{SiO}_4$ ,	$a : b : c = .4584 : 1 : .5791$
Magnesite, $\text{MgCO}_3$ ,	$a : c = 1 : 0.8095$		
Smithsonite, $\text{ZnCO}_3$ ,	$a : c = 1 : 0.8062$		

**Isomorphous Mixtures.**—Since isomorphous compounds have the same crystallographic form, *they naturally tend to crystal-*

*lize together when present in the same crystallizing solution.* The result of this crystallization is a mixture of the different compounds constituting individual crystals, which to all tests appear homogeneous throughout their entire masses when the volume of the crystallized substance that separates is small as compared with the volume of the solution. Mixtures of this kind are called isomorphous mixtures. They are very common among minerals, perhaps more common among the silicates than are simple compounds.

*Isomorphous compounds may be defined from this point of view as those which may crystallize together or which may unite in various proportions to produce homogeneous crystals.* The proportions of the substances in the mixed crystals bears no fixed relation to their molecular weights, as is the case in molecular compounds. On the other hand, they may occur in practically any proportion, depending upon the composition of the solution from which the crystals separate. In some cases there is manifested a tendency for the substances to crystallize together in certain proportions rather than in others, but in many other cases they may crystallize in all proportions possible.

The property of forming mixed crystals is regarded as so characteristic a feature of isomorphous substances that substances are not generally regarded as isomorphous until they have been made to crystallize together or have been found in crystals in various proportions which are not in the ratio of their molecular weights.

**Illustration of Properties of Isomorphous Mixtures.**—An instructive illustration of the dependence of the physical and morphological properties of substances upon their chemical composition is afforded by the crystals composed of mixtures of  $\text{MgSO}_4 + 7\text{H}_2\text{O}$  and  $\text{Zn SO}_4 + 7\text{H}_2\text{O}$ . These compounds are known as *epsomite* and *goslarite* when found in nature. They crystallize in the sphenoidal division of the orthorhombic system, and they form mixed crystals containing all proportions of the two molecules. The properties of the crystals are determined by the percentage of the magnesium (or the zinc) salt present in them. The following table exhibits these relations for a few of the mixtures investigated:



PROPERTIES OF ISOMORPHOUS MIXTURES OF  
 $\text{MgSO}_4 \cdot 7\text{H}_2\text{O}$  and  $\text{ZnSO}_4 \cdot 7\text{H}_2\text{O}$ .

Per cent. of Mg Molecule	Sp. Gr.	Angle $\infty\text{P} \wedge \infty\text{P}$	Optical Axial Angle
100	1.6760	$89^\circ 25'$	$78^\circ 18' 0''$
74.44	1.7472	$89^\circ 15'$	$76^\circ 42' 47''^*$
57.59	1.7977	$89^\circ 8'$	
35.64	1.8604	$89^\circ 1'$	$74^\circ 3' 45''^*$
18.11	1.9094	$88^\circ 54'$	
0	1.9600	$88^\circ 48'$	$70^\circ 53'$

In general the physical properties of crystals composed of mixtures of isomorphous compounds are functions of the compounds crystallizing together, being intermediate between the two pure compounds. This relationship between the physical properties and the chemical composition of mixed crystals is so close that Retgers declares that "*Two substances are truly isomorphous only when the physical properties of their mixed crystals are continuous functions of their chemical composition.*"

**Formulas of Isomorphous Mixtures.**—Although isomorphous mixtures are not definite combinations of elements in the same sense as are simple compounds, nevertheless, they exist so frequently that some means must be decided upon for the representation of their composition. The logical method of writing their formulas is to designate the number of molecules of each of the substances entering into the mixture. This method, however, would require a different formula for every different mixture possible. Since the number of these is often large, a great number of formulas would be demanded, and some of them would be very complicated.

*In practice it is deemed sufficient to indicate by the formula the nature of the molecules in the mixture without specifying exactly their proportions.* This is done by enclosing the symbols of the mutually replaceable elements or groups of elements in parentheses with a comma between them, and writing the symbols of the remaining elements in the usual manner. Such a formula, as, for instance,  $(\text{Fe}, \text{Mg})_2\text{SiO}_4$ , indicates that iron and magnesium

\*Approximate.

silicates are present in different proportions in a series of compounds possessing the same general crystallographic and physical features.

#### ILLUSTRATIONS OF FORMULAS OF ISOMORPHOUS MIXTURES.

Spinel is  $\text{MgAl}_2\text{O}_4$ . Iron spinel is  $\text{MgFe}_2\text{O}_4$ . An isomorphous mixture of these is represented by  $\text{Mg}(\text{Fe},\text{Al})_2\text{O}_4$ .

Barite =  $\text{BaSO}_4$ . Celestite =  $\text{SrSO}_4$ . Barito-celestite =  $(\text{Ba},\text{Sr})\text{SO}_4$ . Tetrahedrite =  $(\text{Ag}_2,\text{Cu}_2,\text{Fe},\text{Zn},\text{Hg})_4(\text{Sb},\text{As})_2\text{S}_7$ .

**Isodimorphous Groups.**—A series of isomorphous polymorphs is an isopolymorphous group. Series of isomorphous compounds each one of which is a dimorph are very common. Such groups are known as isodimorphous groups. A simple example is the following:

<i>Regular.</i>		<i>Orthorhombic.</i>
Arsenolite,	$\text{As}_2\text{O}_3$	Claudetite
Senarmonite,	$\text{Sb}_2\text{O}_3$	Valentinite

**Double Salts.**—Many crystallized substances that are not isomorphous mixtures appear to consist of a combination of molecules which, however, unite in definite proportions, and not in many different proportions like the molecules in isomorphous mixtures. They are known as double salts because they appear to be made up of portions that may exist independently as simple compounds. An illustration in case is sodium silver chloride, whose formula is  $\text{NaCl} + \text{AgCl}$ , or cryolite, a monoclinic mineral with the composition  $\text{Na}_3\text{AlF}_6$  or  $3\text{NaF} \cdot \text{AlF}_3$ . Substances containing water of crystallization also belong to this class of molecular compounds. Gypsum,  $\text{CaSO}_4 + 2\text{H}_2\text{O}$ , is an example.

It is not at all certain that the double salts differ in any essential respect from ordinary atomic molecules. They are referred to here only for the purpose of emphasizing the fact that isomorphous compounds form mixed crystals in which the proportions of the components present may vary with varying conditions during growth, whereas the crystals of a double salt the components of which are not members of an isomorphous series have a definite composition which is invariable.



# INDEX.

---

## A

- Absorption of light, 199-202
  - Acicular, 134
  - Aggregates, 143-147
    - botryoidal, 146-147
    - cryptocrystalline, 144
    - crystalline, 143, 144-147
    - fibrous, 146
    - globular, 145
    - granular, 145
    - lamellar, 145
    - radial, 145
    - sheaf-like, 147
  - Allochromatic, 197, 200
  - Amorphous substances, 6, 160, 177, 180, 214
  - Analogue pole, 216
  - Antilogue pole, 216
  - Axes, crystallographic, 17
    - of hexagonal system, 55-56
    - of isometric system, 32-33
    - of monoclinic system, 116
    - of orthorhombic, 102-103
    - of regular system, 32-33
    - of tetragonal system, 88
    - of triclinic system, 125
    - projection of, 169-173
  - Axial angle, 207-208
    - apparent, 208
    - true, 207
  - ratio, determination of, in hexagonal system, 57-60
  - determination of, in tetragonal system, 89-90
  - of hexagonal system, 56-57
  - of monoclinic system, 116-117
  - of orthorhombic system, 103-104
  - of tetragonal system, 89-90
  - of triclinic system, 126
- Axis, twinning, 152

## B

- Basal pinacoid, 64, 119
- Biaxial crystals, 206
- Boiling point, 211
- Botryoidal, 146, 147
- Brachyaxis, 103, 126
  - domes, 108, 128
  - hemidomes, 128
  - hemiprisms, 127-128
  - pinacoid, 109, 128-129
  - prisms, 107-108, 127-128
  - pyramid, 107-108, 126-127
  - series, 107-108
  - tetrapyrramids, 127

## C

- Chemical compounds, 227
- Classes of symmetry:
  - dihexagonal bipyramidal, 60-69
  - ditetragonal bipyramidal, 87-95
  - ditrigonal bipyramidal, 78-80
  - ditrigonal pyramidal, 80-81
  - domatic, 121-122
  - dyakisdodecahedral, 46-48
  - hexagonal bipyramidal, 75-76
  - hexagonal trapezohedral, 76-77
  - hexoctahedral, 31-38
  - hextetrahedral, 48-51
  - orthorhombic bipyramidal, 105-110
  - orthorhombic bisphenoidal, 112-114
  - orthorhombic pyramidal, 110-112
  - pentagonal icositetrahedral, 46
  - pinacoidal, 125-131
  - prismatic, 117-121
  - sphenoidal, 122-124
  - tetragonal bipyramidal, 98-99
  - tetragonal scalenohedral, 95-98
  - tetragonal trapezohedral, 100
  - trigonal scalenohedral, 70-74
  - trigonal trapezohedral, 82-86

- Cleavage, 181-183
    - planes of, 181-182
    - symmetry of, 182-183
  - Clinoaxis, 116
    - domes, 119
    - hemipyramids, 118-119
    - pinacoids, 119-120
    - prisms, 119
  - Closed forms, 68, 109
  - Coefficient of elasticity, 180
    - of expansion, 210-211
  - Cohesion, 181
  - Colloids, 6, 8, 160
  - Color, 196-197
    - allochromatic, 197, 200
    - ideochromatic, 197, 200
  - Columnar, 134
  - Combinations, 43-44
    - hexagonal, 68-69, 73-74, 80-81, 85-86
    - isometric, 38-39, 51-53
    - monoclinic, 120-121
    - orthorhombic, 108, 109-110
    - tetragonal, 94-95, 99-100
    - triclinic, 129-130
  - Composition face, 152
  - Conchoidal fracture, 187-188
  - Conduction of electricity, 215
    - of heat, 209-210
  - Congruent forms, 47
  - Constancy of interfacial angles, 14-16
  - Contact twins, 152-153
  - Contraction in crystals, 210-211
  - Corrosion of planes, 136
  - Cryptocrystalline, 144
  - Crystal angle, 10
    - axes, 17
    - biaxial, 206
    - definition of, 8
    - drawing, 168-174
    - edge, 10
    - form, 21, 27
    - group, 143
    - inclusions, 141
    - individual, 8
    - liquid, 8
    - particles, 4-5
    - projection, 164-174
  - Crystal, structure of, 5-7
    - uniaxial, 206
  - Crystalline bodies, 6-9, 177
  - Crystallites, 141
  - Crystallization, 8-9
  - Crystallographic axes, 17
    - constants of monoclinic system, 116-117
    - of triclinic system, 126
  - Crystallography, 9
    - definition of, xi
    - laws of, 9-10
    - systems of, 28, 29-30
    - comparison of, 130-131
  - Cube, 37
  - Curvature of planes, 136
  - Cyclic twins, 155
- D
- Dana's notation, 22
  - Deformed crystals, 134-135
  - Dendritic growth, 149
  - Density. See specific gravity.
  - Determination of axial ratio in tetragonal system, 89-90
  - in hexagonal system, 57-60
  - Diamagnetic, 214
  - Diathermous, 209
  - Dichroscope, 201
  - Dichroism, 201
  - Dihexagonal bipyramid, 61-62
    - prism, 64
    - series, 62-64
  - Dimorph, 162
  - Dimorphous substances, 227-229
  - Diploid, 47
  - Distorted crystals, 132-133
  - Ditetragonal bipyramid, 90-91
    - series, 91-92
  - Ditrigonal bipyramids, 79-80
    - prisms, 79-80
    - of second order, 84
  - Dodecahedron, 36
  - Double refraction, 204-206
    - explanation of, 205-206
  - Double salts, 233
  - Drawing of crystals, 168-174

Druse, 149-150  
Dyakisdodecahedron, 47

E

Elasticity, 180  
Electrical properties, 214-218  
Electricity, conduction of, 215  
Enantiomorphous forms, 83  
Etched figures, 136, 220-223  
    symmetry of, 221-223  
Expansion of crystals, 210-211  
Extraordinary ray, 205

F

Fibrous aggregates, 146  
Fluid inclusions, 139, 142  
Fluorescence, 202  
Fossilization, 163  
Fowlings, 154, 157  
Fracture, 186-187  
Fusibility, scale of, 212-213  
Fusing points, 211  
    table of, 212  
Fusion, 211-212

G

Gas inclusions, 139-142  
Glass inclusions, 141  
Gliding planes, 183-184  
Globular aggregates, 145  
Goniometer, 11-13  
Granular aggregates, 145  
Groundform, 33  
Gyroidal hemihedrism, 46

H

Habit of crystals, 14, 133-134  
    acicular, 134  
    columnar, 134  
    prismatic, 134  
    tabular, 134  
Hardness, 187-192  
    curves of, 190-191  
    determination of, 189  
    differences in, 189, 190  
    list of relative, 188  
    scale of, 187  
Haüy, x

Heat, conduction, 209-210  
    transmission, 209

Heavy solutions, 194

Klein, 194  
Retgers, 194  
Rohrbach, 194  
Thoulet, 194

Hemibrachydomes, 128

Hemiclinodomes, 122

Hemihedrism, 31, 342  
    in hexagonal system, 69-82  
    in isometric system, 41-53  
    in monoclinic system, 123-124  
    in orthorhombic system, 112-114  
    in tetragonal system, 95-101  
    law of, 42-43

Hemimacrodomes, 128

Hemimorphism, 31, 41-42  
    in hexagonal system, 80-81  
    in monoclinic system, 121-123  
    in orthorhombic system, 110-112  
    in tetragonal system, 101

Hemiprisms, 122, 127-128

Hemipyramids, 118

Hexagonal prisms of first order, 65-66  
    of second order, 66-67  
    pyramids of first order, 65-66  
    of second order, 66-67  
    rhombohedral, 71-73  
    scalenohedrons, 71  
Hexagonal system, 54-86  
    axes of, 55-56  
    axial ratio in, 56-57  
    determination of, 57-60  
    combinations in, 68-69, 73-74  
    comparison with tetragonal system,  
        54-55, 87

    general form in, 61  
    groundform in, 56  
    hemihedral division, 69-82  
    hemimorphism in, 80-81  
    holohedral division, 60-69  
    symmetry of, 54-55  
    tetartohedrism in, 82-86  
    three series of holohedrons in, 64-65  
    trapezohedrons, 77

Hexoctahedron, 34

Hextetrahedron, 49

Holohedron, 31

Hopper-shaped crystals, 137

## I

Icositetrahedron, 35

Ideal forms, 132

Ideochromatic, 197, 200

Idiomorphic form, 3, 4

Imperfections in crystals, 132-142

Inclusions, 138-142

crystal, 141

fluid, 139, 142

gas, 139, 141, 142

glass, 140

Index of refraction, 203-204

Indices, 23

Intercept on lateral axes, hexagonal system, 58-60

Interfacial angles, 10, 11

constancy of, 14-16

measurement of, 11-13

edge, 10

Interpenetration twins, 153, 154, 156, 157, 158

Irregularities in crystals, 132

Isodimorphous group, 233

Isometric system, 31-53

axes of, 32-33

combinations in, 38-39, 43-44

derivation of forms in, 34

determination of forms in, 37-38

general form in, 33-34

groundform in, 33

hemihedral division, 41-53

holohedral division, 31-40

symmetry of 31-32

tetartohedrism in, 53

Isomorphism, 227, 229-230

Isomorphous compounds, 230-231

formulas of, 232-233

properties of, 231-232

groups, 230

mixtures, 230-231

Isotherms, 210

## J

Jolly balance, 192, 193

## K

Klein solution, 194

## L

Lamellar aggregates, 145

Law of constancy of interfacial angles, 14-16

of hemihedrism, 42-43

of rationality of the indices, 17

of simple mathematical ratios, 17-20

application of, 22

of symmetry, 28

of tetartohedrism, 42-43

Light, absorption of, 199

reflection of, 196

refraction of, 203-208

transmission of, 199

Liquid crystals, 8

List of fusing points, 212

of hexagonal hemihedrons, 81

of isometric hemihedrons, 51

of isodimorphous groups, 230

of isomorphous groups, 230

of polymorphous bodies, 228-229

of specific gravities, 195

of tetragonal hemihedrons, 101

Luster, 198, 199

## M

Macroaxis, 103, 126

domes, 108-109, 128

pinacoid, 109, 128-129

prism, 107, 127-128

pyramids, 107, 126-127

series, 106

tetrapyramids, 127

hemiprisms, 127-128

hemidomes, 128

Magnetic properties of crystals, 214

Magnetism, 214

Mechanical properties of crystals, 180-195

Merohedrism, 41

Microlites, 141

Miller's system of notation, 23-24

Mimicry, 158

Mineral, definition of, ix

- Mineralogy, history of, x  
 purpose of, ix  
 subdivisions of, xi-xii
- Mohs's scale of hardness, 187
- Molecules, 4-6, 7-8
- Monoclinic domes, 19  
 hemiclinodomes, 122  
 hemipyramids, 118-119  
 orthodomes, 119  
 pinacoids, 119-120  
 prisms, 119  
 system, 115-124  
   axes of, 116  
   combinations in, 120-121  
   crystallographic constants in, 116-117  
   groundforms in, 116-117  
   hemihedral division, 123-124  
   hemimorphism in, 121-123  
   holohedral division, 117-121  
   symmetry of, 115-116  
   tetraorthodomes, 122  
   tetrapyramids, 122
- Morphological mineralogy, 9
- Morphotropism, 16, 230
- N
- Naumann's system of notation, 21-22, 24
- Negative crystals, 139
- Notation, crystallographic, 20  
   Miller's system of, 23-24  
   Naumann's system of, 21-22, 24  
   Weir's system of, 20-21
- O
- Octahedron, 33, 37
- Open forms, 68
- Optical axes, 207  
   properties of crystals, 196-208
- Ordinary ray, 205
- Orthorhombic brachydomes, 108-109  
   pinacoids, 108-109  
   prisms, 107-108  
   pyramids, 107-108  
   macrodomes, 108-109  
   pinacoids, 108-109  
   prisms, 106-107
- Orthorhombic pyramids, 106-107  
   system, 102-114  
     axes of, 102-104  
     closed forms in, 109  
     combinations in, 108, 109-110  
     general form in, 105  
     groundform in, 103  
     hemihedral division, 112-114  
     hemimorphism in, 110-111  
     holohedral division, 105-112  
     symmetry of, 102  
     three series of forms in, 105  
   unit prisms, 105-106  
   prisms, 105-106
- P
- Parallel growths, 143, 147-151
- Paramagnetic substance, 214
- Parameters, 18-19
- Paramorphs, 162
- Partial forms, 41
- Parting, 186
- Penetration twins, 153, 154
- Pentagonal dodecahedrons, 47-48  
   hemihedrons, 46-48  
   icositetrahedron, 46
- Percussion figures, 185, 186
- Phanerocrystalline, 144
- Phosphorescence, 202-203
- Physical agencies, 178-179
- Physical symmetry, 177, 178
- Plane of symmetry, 28
- Pleochroism, 200-202
- Polymorphism, 227-229
- Polymorphous bodies, 227, 229
- Polysynthetic twins, 155, 156
- Pressure figures, 186
- Projection, crystal, 164-174  
   linear, 164-167, 168  
   of axes, 169-173  
     isometric, 170  
     tetragonal, 170  
     orthorhombic, 170-171  
     monoclinic, 171  
     triclinic, 171-172  
     hexagonal, 172-173  
   spherical, 168, 167



- Pseudomorphism, explanation of, 161  
Pseudomorphs, 160-163  
    chemical, 162  
    mechanical, 162-163  
Pyramidal hemihedrisms, 70, 75-76, 95, 98-99  
Pyroelectrical properties of crystals, 216-218  
Pyroelectricity, 216  
Pyritohedron, 47-48
- R**
- Radial aggregates, 145  
Re-entrant angle, 152  
Reflection of light, 196  
Refraction, double, 204-206  
    single, 204  
    of light, 203-208  
    index of, 203-204  
Relation between hexagonal pyramids and prisms of various orders, 67-68, 76  
    tetragonal pyramids and prisms of various orders, 94  
Repeated twins, 154-158  
Retgers solution, 194  
Rhombohedral hemihedrisms, 70-74  
Rhombohedrons, 71-72  
    short symbols of, 73  
Rock, definition of, ix  
Rohrbach solution, 194
- S**
- Scale of fusibility, 212-213  
    hardness, 187  
Scalenoedrons, 71, 96-97  
Secondary twinning, 184  
Sheaf-like aggregates, 147  
Short symbols of rhombohedra and scalenoedra, 73  
Simple mathematical ratios, 17  
Simple refraction, 204  
Sklerometer, 189  
Solution, 219-223  
    planes of, 219-220  
    rates of, 219  
    symmetry of, 220
- Specific gravity, 192-195  
    determination of, 192-195  
    list of, 195  
    use of, 195  
Sphenoidal hemihedrisms, 95-98, 112-114  
Sphenoids, 97-98, 113-114  
Stalactites, 3-4, 147  
Streak, 197  
Striations, 135-136, 137-138  
Structure of crystals, 5-7  
Sub-individuals, 149  
Supplementary twins, 153-154  
Symbols of planes, 18-24  
Symmetry, 25-30  
    axes of, 25  
    classes of, 29  
    grades of, 28  
    in crystallography, 26-28  
    law of, 28  
    of crystal faces, 178  
    of etched figures, 221-223  
    of pyroelectrical properties, 216-218  
    of solution, 220  
    physical, 177-178  
    planes of, 28  
    with respect to lines, 25-26  
    to planes, 25  
    to points, 26  
Systems, crystallographic, 28, 29-30
- T**
- Table of relative hardness, 188  
    of fusing points, 212  
    of hexagonal hemihedrons, 81  
    of isometric hemihedrons, 51  
    of tetragonal hemihedrons, 101  
Tenacity, 180-181  
Tetartohedrisms, 31, 42  
    in hexagonal system, 82-86  
    in isometric system, 42, 53  
    in tetragonal system, 101  
    law of, 42-43  
Tetrabachypyramids, 126-127  
Tetragonal bipyramids of first order, 92-93  
    of second order, 93-94  
    of third order, 98-99

